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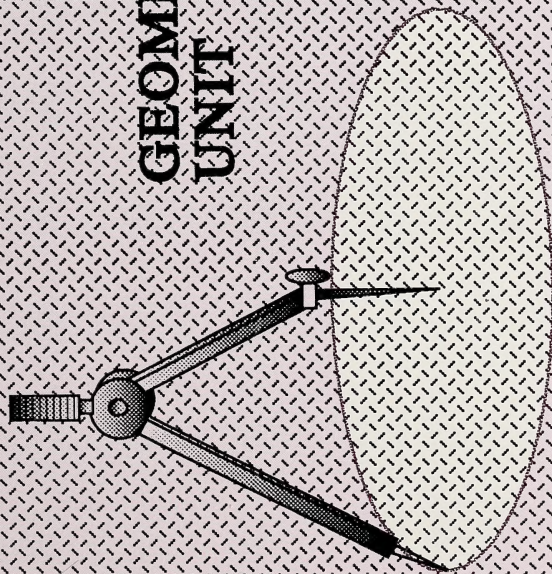
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MATHEMATICS 23

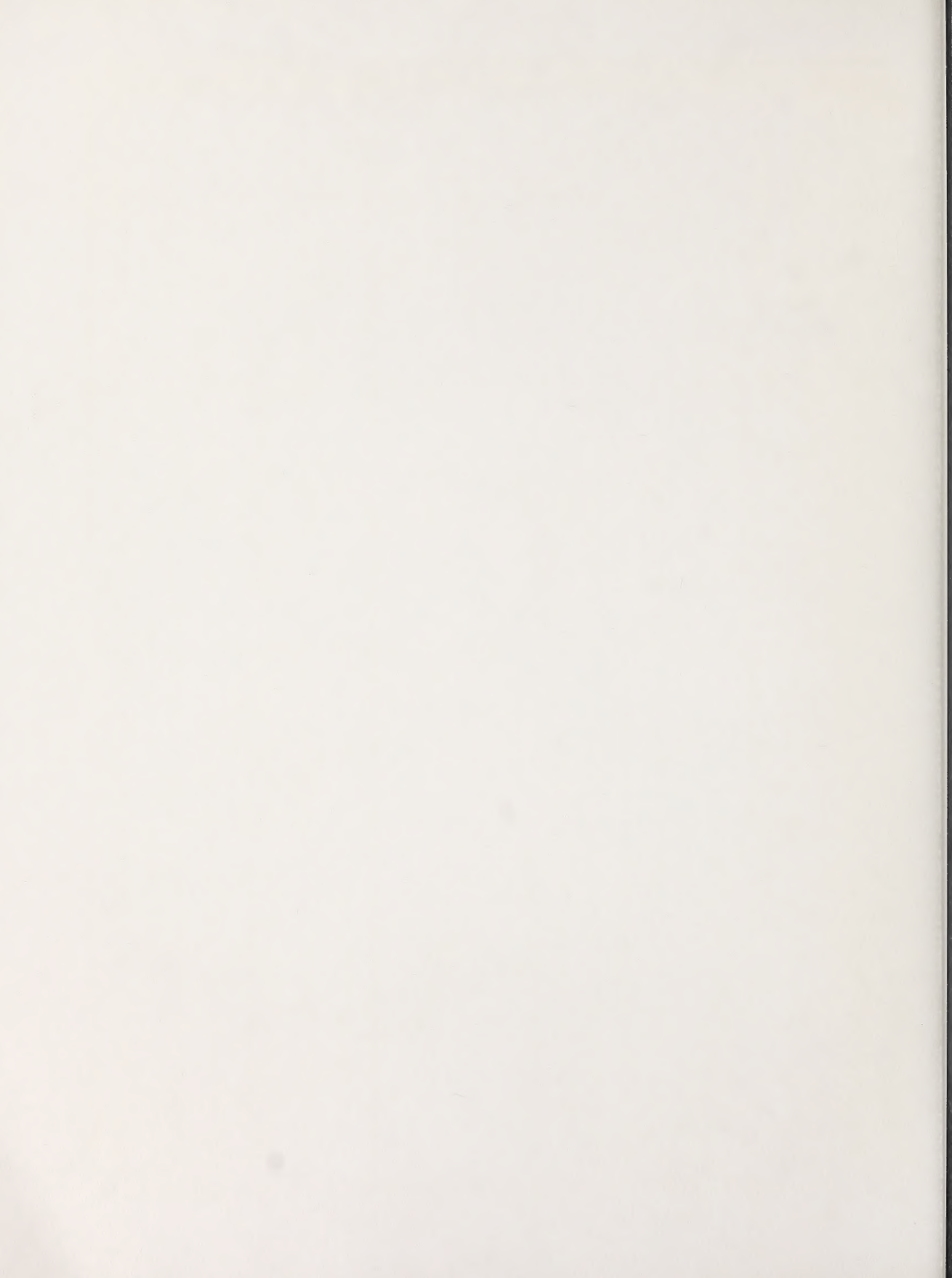
GEOMETRY UNIT 6



Alberta
EDUCATION

**Distance
Learning**





W e l c o m e



Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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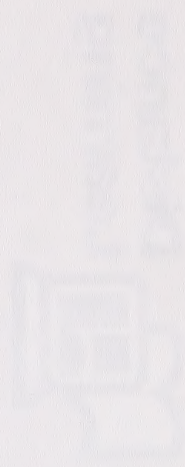
Mathematics 23 Student Module Unit 6 Geometry Alberta Distance Learning Centre ISBN No. 0-7741-0006-0 *1992

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WELCOME

General Information

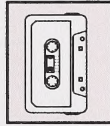
This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics (**Appendix B, etc.**).

Visual Cues

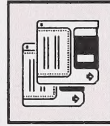
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.



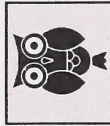
Audiotape

- learning by listening to an audiotape



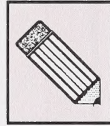
Computer Software

- learning by using computer software



What You Already Know

- reviewing what you already know



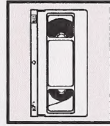
Review

- studying previous concepts



Solutions

- correcting the activities



Videotape

- learning by viewing a videotape



Introduction

- introducing the unit



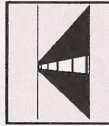
Extra Help

- providing additional study



Print Pathway

- choosing a print alternative



What Lies Ahead

- previewing the unit



Extensions

- going on with the topic



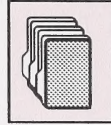
Calculator

- using your calculator



Exploring the Topic

- actively learning new concepts



What You Have Learned

- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing the unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit assignment - 50%
Supervised unit test - 50%

Introduction to Geometry

This unit covers topics dealing with Geometry. Each topic contains explanations, examples, and activities to assist you in understanding geometry. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in **Appendix A**. In several cases there is more than one way to do the question.

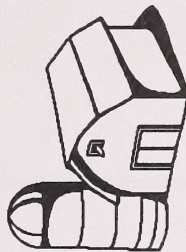
Unit 6 Geometry

Contents at a Glance

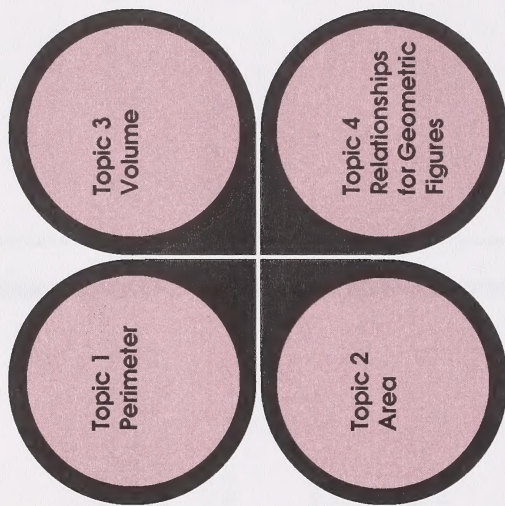
Value	Geometry	3
	What You Already Know	5
	Review	6
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	• Introduction	
	• What Lies Ahead	
	• Exploring Topic 1	
	• Extra Help	
	• Extensions	
35%	Topic 2: Area	27
	• Introduction	
	• What Lies Ahead	
	• Exploring Topic 2	
	• Extra Help	
	• Extensions	
35%	Topic 3: Volume	70
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	• Extensions	
12%	Topic 4: Relationships for Geometric Figures	94
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Geometry

In geometry, you study the size, shape, and other properties of figures. In this unit on geometry you will study not only plane figures, but also those that exist in three dimensions and take up space. After studying this unit, you should be able to calculate how much silage can be stored in a silo.



Unit 6 Geometry





What You Already Know

Refresh your memory!

- When using a calculator, you can find approximations for square and cube roots.

- You can factor trinomials of the form $x^2 + bx + c$ by inspection.

$$x^2 + 4x - 21 = (x - 3)(x + 7)$$

$$x^2 + 11x + 18 = (x + 9)(x + 2)$$

Now that you have looked at material that you studied previously, go to the **Review** to confirm your understanding of this material.

You can find the square root of 3 to two decimal places.



Enter 3

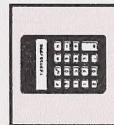
Press

The resulting display is 1.732050808.

This rounds to 1.73.

The square root of 3 is 1.73.

You can determine $\sqrt[3]{25}$ to two decimal places.



Enter 25

Press

The resulting display is 2.924017738.

This rounds to 2.92.

The cube root of 25 is 2.92.

The cube root function may be a second function on your calculator. Then press or and the appropriate key.



Review

Try the following review questions.

1. Calculate the value of the following to two decimal places.

- a. $\sqrt{85}$
- b. $\sqrt{96}$
- c. $\sqrt[3]{30}$
- d. $\sqrt[3]{941}$

2. Factor the following trinomials.

- a. $x^2 - 18x - 63$
- b. $x^2 + x - 20$



Now go to the **Review** solutions in **Appendix A**.

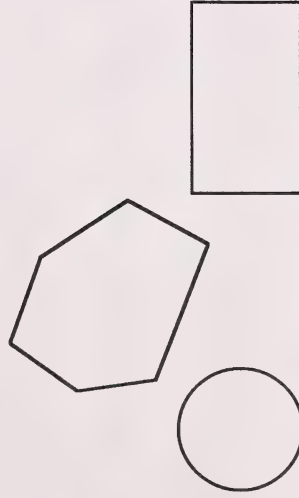
Topic 1 Perimeter



Introduction

The perimeter is the distance around a geometric figure. For a polygon, the perimeter is equal to the sum of the side lengths. For a circle, the perimeter is equal to the circumference.

In this topic, you will learn to find the perimeter of figures with a variety of shapes.



What Lies Ahead

Throughout this topic you will learn to

1. determine the perimeter of a polygon
2. develop and use the formula for the perimeter of a regular polygon
3. determine and use the fact that π is the ratio between the circumference and diameter of a circle
4. solve problems involving the perimeter of closed figures

Now that you know what to expect, turn the page to begin your study of perimeter.



Exploring Topic 1

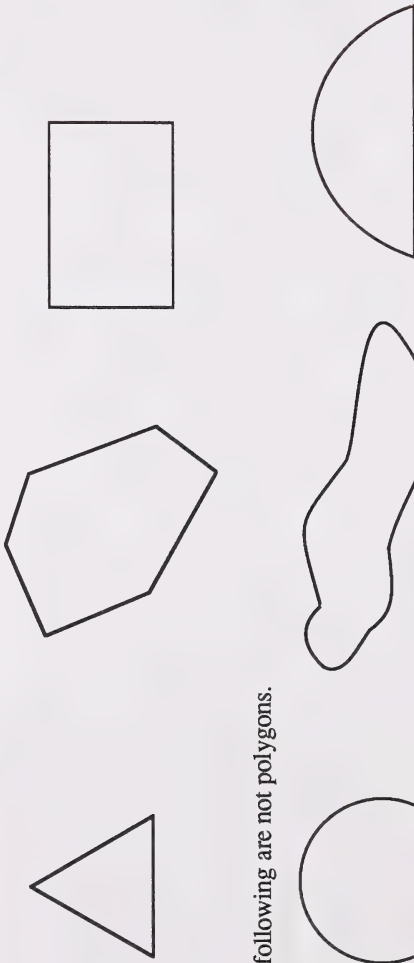
Activity 1



Determine the perimeter of a polygon.

A **polygon** is a closed figure made up of line segments. The line segments are called sides of the polygon.

The following are examples of polygons.



The following are not polygons.

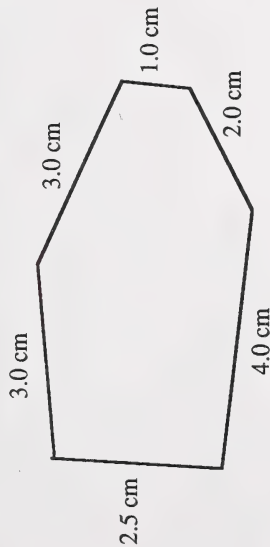
The boundary of closed figures that are not polygons have curved parts.

For polygons, the perimeter is equal to the sum of the lengths of the sides.

Look at the following examples.

Example 1

Determine the perimeter of the polygon.



Solution:

The sum of the side lengths is $2.5 \text{ cm} + 3.0 \text{ cm} + 3.0 \text{ cm} + 1.0 \text{ cm} + 2.0 \text{ cm} = 15.5 \text{ cm}$.

The perimeter of the polygon is 15.5 cm.

Example 2

Determine the perimeter of the triangle with side lengths as shown.



Solution:

The sum of the side lengths is $39 \text{ m} + 31 \text{ m} + 59 \text{ m} = 129 \text{ m}$.

The perimeter of the triangle is 129 m.

In general, the perimeter of a triangle with side lengths a , b , and c is as follows:

$$P = a + b + c$$



Now look at an example involving the perimeter of a rectangle.

Example 3

What is the perimeter of this rectangular swimming pool?



Solution:

Opposite sides have the same length. The sum of side lengths is $10 \text{ m} + 25 \text{ m} + 10 \text{ m} + 25 \text{ m} = 70 \text{ m}$.

The perimeter of the swimming pool is 70 m.

From the example you can see that for a rectangle with a length of l and a width of w , the perimeter is $P = 2(l + w)$.



The next example shows how this formula can be applied.

Example 4

Each day, Karin Schnell runs around a field once. The field has a length of 540 m and a width of 260 m. How far does she run daily?

Solution:

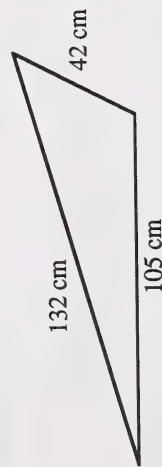
$$\begin{aligned}
 P &= 2(l + w) \\
 &= 2(540 \text{ m} + 260 \text{ m}) \\
 &= 2(800 \text{ m}) \\
 &= 1600 \text{ m}
 \end{aligned}$$

Karin runs 1600 m daily.

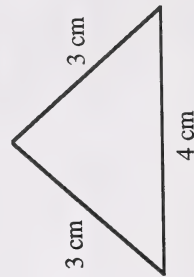
Try the following questions. Do at least one of the following two questions.

1. Determine the perimeter of the figures. Use the side lengths given.

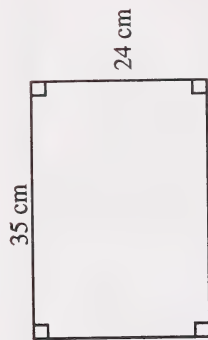
a.



a.

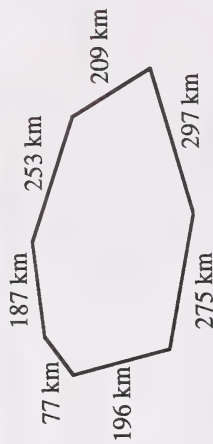


b.



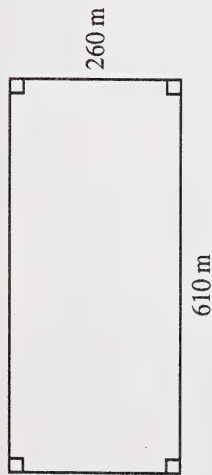
(Note: The symbol $\square$ indicates a 90° angle.)

c.



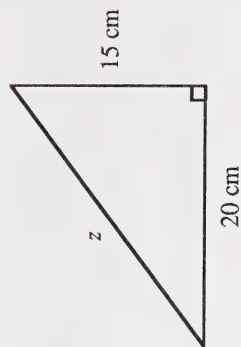
2. Determine the perimeter of each of the figures with the indicated side lengths.

b.



Example 5

Determine the perimeter of the right triangle.



Solution:

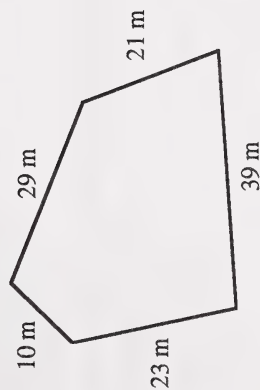
For this triangle, $x^2 + y^2 = z^2$ applies when $x = 20$ and $y = 15$.

$$\begin{aligned} z^2 &= x^2 + y^2 \\ &= 20^2 + 15^2 \\ &= 400 + 225 \\ &= 625 \\ z &= \sqrt{625} \\ &= 25 \end{aligned}$$

The length of the hypotenuse is 25 cm.

The perimeter of the triangle is $20 \text{ cm} + 15 \text{ cm} + 25 \text{ cm} = 60 \text{ cm}$.

c.

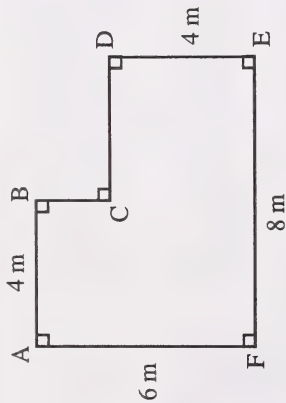


For solutions to Activity 1, turn to **Appendix A, Topic 1**.

You may have to determine the lengths of sides before finding the perimeter. This is shown in the next examples.

Example 6

Determine the perimeter of the figure.



Solution:

$$\begin{aligned} AB + CD &= EF \\ CD &= EF - AB \\ &= 8 \text{ m} - 4 \text{ m} \\ &= 4 \text{ m} \end{aligned}$$

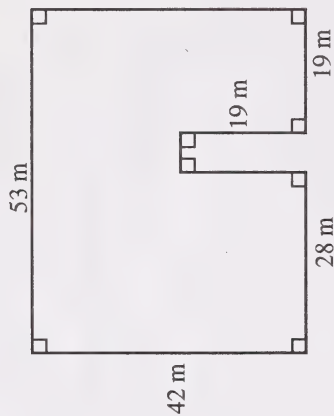
$$\begin{aligned} BC + DE &= AF \\ BC &= AF - DE \\ &= 6 \text{ m} - 4 \text{ m} \\ &= 2 \text{ m} \end{aligned}$$

$$\begin{aligned} P &= AB + BC + CD + DE + EF + AF \\ &= 4 \text{ m} + 2 \text{ m} + 4 \text{ m} + 4 \text{ m} + 8 \text{ m} + 6 \text{ m} \\ &= 28 \text{ m} \end{aligned}$$

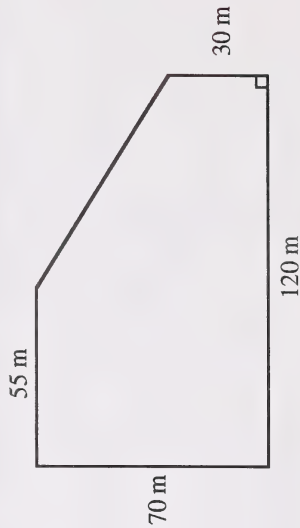
The perimeter of the figure is 28 m.

Try the following questions. Do at least three of the following five questions.

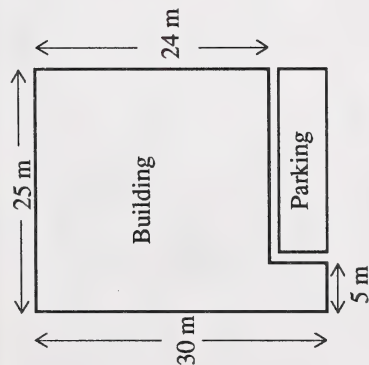
- Determine the perimeter of the right triangle which has a hypotenuse of 5.9 cm and one leg measuring 5.1 cm. Round your answer to one decimal place.
- Determine the perimeter of the figure.



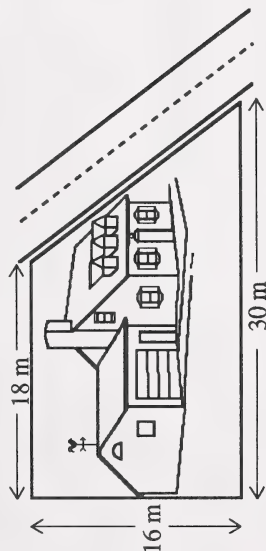
- Determine the perimeter of the figure. Round your answer to the nearest whole metre.



6. What is the perimeter of the base of the building?



7. The Chan family owns a lot as shown. It is an odd shape because one corner is cut off by a street. The Chans want to plant a cotoneaster hedge along the boundary of the lot. If the plants are to be placed every 50 cm, how many plants will be needed?



For solutions to Activity 1,
turn to Appendix A, Topic 1.



Activity 2



Develop and use the formula for the perimeter of a regular polygon.

A regular polygon is a polygon in which angles are equal and sides are equal.

These polygons are regular.



These polygons are irregular.

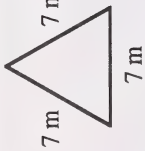
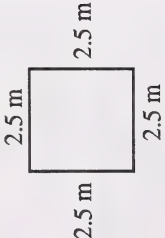
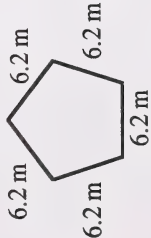
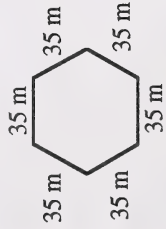


Note: In the rectangle, the angles are equal but the sides are not.
In the other two figures, neither angles nor sides are equal.

For any polygon, you can find the perimeter by adding the lengths of the sides. For regular polygons, it is easier to find this sum because the sides are equal. In fact, you can develop a formula to determine the perimeter of a regular polygon. The next questions may help you do this.

Try the following questions. Use the table for these questions.

1. In the table, fill in the missing entries. Determine the perimeters by adding the side lengths.

Figure (not all drawn to the same scale)	Classification	Number of Sides	Side Length (m)	Perimeter (m)
	Equilateral Triangle	3	7 m	$7\text{ m} + 7\text{ m} + 7\text{ m} = 21\text{ m}$
	Square	4	2.5 m	
	Regular Pentagon			
	Regular Hexagon			

2. For each of the regular polygons, how does the perimeter compare to the side length?
3. Write out a formula for the perimeter of a regular polygon. Let P , n , and s respectively be the perimeter, the number of sides, and the side length.



For solutions to Activity 2, turn to **Appendix A, Topic 1.**

From your experience, you can conclude that for regular polygons the perimeter can be determined with this formula:

$P = ns$, where n is the number of sides, and
 s is the length of a side.

Example 7

Determine the perimeter of an eighteen-sided regular polygon in which each side measures 3.45 m.

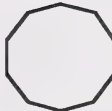
Solution:

$$\begin{aligned} P &= ns \\ &= 18(3.45 \text{ m}) \\ &= 62.1 \text{ m} \end{aligned}$$

The perimeter of the polygon is 62.1 m.

Try the following questions in which you can apply the perimeter formula. Do at least question 4.

4. Fill in the missing entries in the table.

Figure (not drawn to scale)	Classification	Number of Sides	Side Length (m)	Perimeter (m)
	Regular Heptagon	7	21 m	
	Regular Octagon		1.25 m	
	Regular Nonagon	9	12 m	
	Regular Decagon			120 m

5. A dodecagon is a polygon of twelve sides. What is the perimeter of a regular dodecagon with sides of 10.5 cm?
6. A regular polygon has a perimeter of 360 cm. Each side measures 12 cm. How many sides does this polygon have?
7. Each side of a regular polygon measures 35 cm. If the perimeter of this polygon is 385 cm, how many sides does it have?



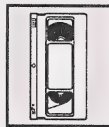
For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

Activity 3



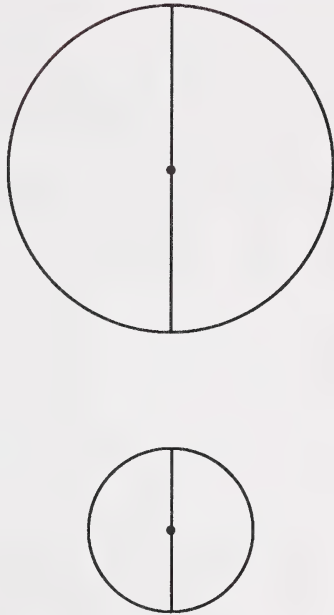
Determine and use the fact that π is the ratio between the circumference and diameter of a circle.

You may view the video titled *Mathways*¹, program three, as an introduction to this activity. Pay close attention to the section on pi (π) and circumference. You may stop the tape after the section on circumference at this point. The remainder of this program may be viewed along with another part of this unit.



¹ *Mathways* is a title of Wisconsin ETV Network.

The perimeter of a circle is called the circumference of the circle. It is related to the diameter of the circle. Circles with larger diameters also have larger circumferences.



There is a direct relationship between the diameter and circumference of a circle. You will discover this in the next questions.

1. For each of several round objects available to you, measure the circumference and the diameter. You will need a flexible tape measure. Some objects you can use are tin cans, plates, and wheels. Complete a table such as that following. Record the circumference and the diameter in the same units. (In the last column, round to two decimal places.)

Object	Circumference (C)	Diameter (d)	$C \div d$

2. Complete the last column in the table. Round to two decimal places.

Object	Circumference (C)	Diameter (d)	$C \div d$
trailer wheel	140.0 cm	44.5 cm	
soup can	23.2 cm	7.4 cm	
fruit can	31.4 cm	10.0 cm	
vegetable can	45.6 cm	14.5 cm	
plate	73.8 cm	23.6 cm	
saucer	40.0 cm	12.7 cm	

3. By what factor is the circumference larger than the diameter?



For solutions to Activity 3, turn to Appendix A, Topic 1.

You have discovered that the circumference is close to 3.14 times as large as the diameter. The ratio of circumference to diameter is represented by the Greek letter π which is written as π . The value of π can only be approximated by a terminating decimal.

$$\pi = 3.141\,592\,654 \dots$$

For this course, you can use the rounded value of 3.14 for π if your calculator does not have a π key.

The value π is the ratio of circumference to diameter. That is

$$\pi = \frac{C}{d}.$$

When you solve this equation for C , you obtain

$$C = \pi d.$$

Since the diameter is twice the radius, the following also holds:

$$C = 2\pi r$$

Look at the examples to see how π can be used.

Example 8

Glen Yakimovich found that his watch had a face with a diameter of 3.1 cm. What is the circumference of the watch face?



Solution:

$$\begin{aligned} C &= \pi d \\ &= \pi (3.1 \text{ cm}) \\ &= 9.7 \text{ cm} \end{aligned}$$

The circumference of the watch face is 9.7 cm.

Try the following questions. Do at least three of the five questions.

4. When the tires are inflated, the diameter of each of Sandy's bicycle wheels is 77 cm. Sandy rides her bike to school which is 1.5 km away from her home. How many turns must each of the wheels make on the way to school? Round to the nearest revolution.

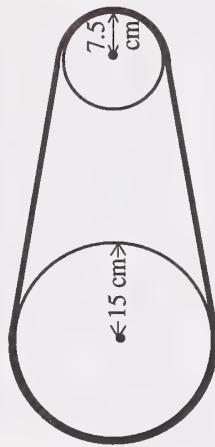
Some calculators have a π key. You can use this key to obtain a very good approximation for π .

Hint: Express all values.

5. The circle was made with a compass separation of 2.1 cm. Calculate the circumference of the circle.



8. The wheels below are connected by a belt. If the larger wheel makes one complete rotation, how many times will the smaller wheel turn?



For solutions to Activity 3, turn to Appendix A, Topic 1.



Activity 4



Solve problems involving the perimeter of closed figures.

7. John placed a mug of tea on a table. Unfortunately, this left a stain. The stain was circular with a diameter of 5.6 cm. What was the circumference of this stain?

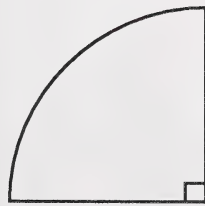
Polygons and circles are closed figures. However, some closed figures fit neither category; yet their perimeters can be calculated from your understanding of perimeter for polygons and circles.

This is shown in the next examples.

Hint: Compare the circumference of the wheels.

Example 9

The figure is drawn to scale. Determine its perimeter.



Solution:

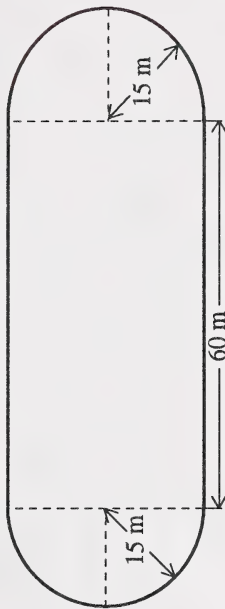
Each straight side measures 3.0 cm. The curved portion is $\frac{1}{4}$ of a circle of radius 3.0 cm. The length, l , of this curved portion is found using the following calculation.

$$\begin{aligned} l &= \left(\frac{1}{4}\right)(2\pi r) \\ &= \left(\frac{1}{2}\right)(\pi)(3.0 \text{ cm}) \\ &= 4.7 \text{ cm} \end{aligned}$$

The perimeter is $3.0 \text{ cm} + 3.0 \text{ cm} + 4.7 \text{ cm} = 10.7 \text{ cm}$.

Example 10

Suppose a running track was built as shown. How far would you have gone if you ran around the track ten times?



Solution:

Each of the end curves makes a semicircle. The length of the track equals the circumference of a circle of radius 15 m plus the length of the two straight sections each measuring 60 m.

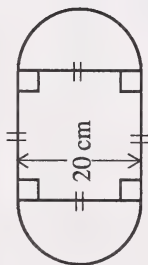
$$\begin{aligned} \text{The track length equals } 2(60 \text{ m}) + 2\pi(15 \text{ m}) &= 120 \text{ m} + 94 \text{ m} \\ &= 214 \text{ m.} \end{aligned}$$

When going around ten times, you would run

$$\begin{aligned} 10(214 \text{ m}) &= 2140 \text{ m} \\ &= 2.140 \text{ km.} \end{aligned}$$

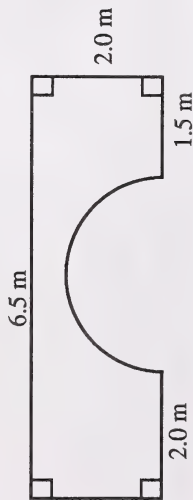
Try the following questions. Do at least three of the five questions.

- Find the perimeter of the figure.

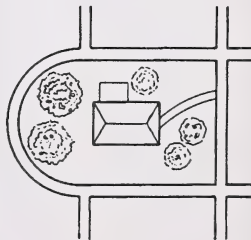


(Note: The symbol $\perp$ indicates a 90° angle.)

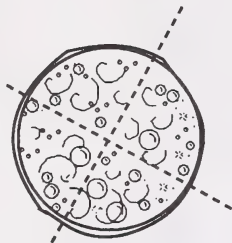
- Calculate the perimeter of the following.



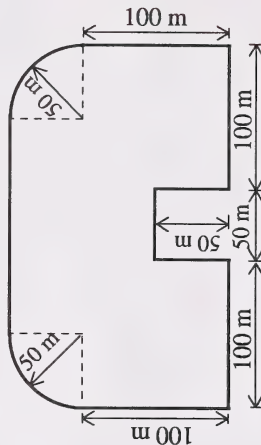
- The lot pictured is bound by three straight sides each measuring 20 m. The remaining part of the boundary forms a semicircle. What is the distance around the lot?



- Suppose a pizza with a diameter of 25 cm is cut into four equal pieces as shown. What is the perimeter of each piece?



- From the floor plan of a public building, determine the perimeter. The top corners are parts of a circle with radius 50 m.



For solutions to Activity 4, turn to Appendix A, Topic 1.

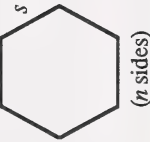
If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

Perimeter is the distance around a figure.

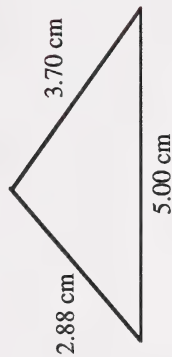
Figure	Formula for Perimeter
 Triangle	$P = a + b + c$
 Rectangle	$P = 2(l + w)$
 Circle	$C = \pi d$
 Regular Polygon	$P = ns$

Look at the example.

Example 11

Determine the perimeter of the following:

- a triangle as shown in the diagram



Solution:

$$\begin{aligned}
 P &= a + b + c \\
 &= 2.88 \text{ cm} + 5.00 \text{ cm} + 3.70 \text{ cm} \\
 &= 11.58 \text{ cm}
 \end{aligned}$$

The perimeter of the triangle is 11.58 cm.

- a parallelogram as shown in the diagram



Solution:

$$\begin{aligned}
 P &= 2(l + w) \\
 &= 2(5.60 \text{ m} + 2.40 \text{ m}) \\
 &= 2(8.00 \text{ m}) \\
 &= 16.00 \text{ m}
 \end{aligned}$$

The perimeter of the parallelogram is 16.00 m.

- a circle with a diameter of 12.5 cm

Solution:

$$\begin{aligned}
 C &= \pi d \\
 &= (\pi)(12.5 \text{ cm}) \\
 &= 39.3 \text{ cm}
 \end{aligned}$$

The perimeter of the circle is 39.3 cm.

- a regular decagon with side lengths of 9.6 cm

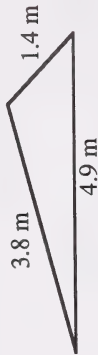
Solution:

$$\begin{aligned}
 P &= ns \\
 &= 10(9.6 \text{ cm}) \\
 &= 96 \text{ cm}
 \end{aligned}$$

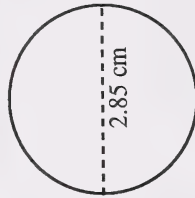
The perimeter of the regular decagon is 96 cm.

Try the following questions.

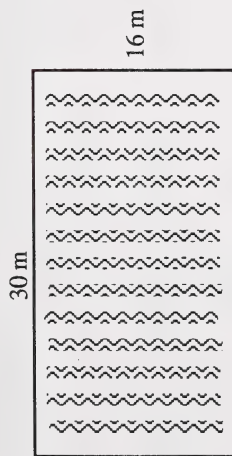
1. What is the perimeter of the triangle?



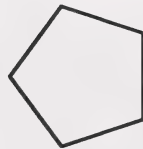
2. Determine the circumference of the circle.



3. Determine the perimeter of the garden.



4. The perimeter of this regular polygon is 125 cm. What is the length of each side?

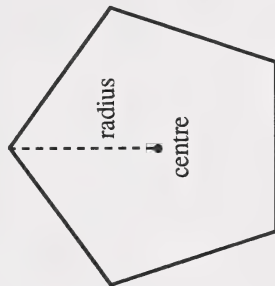


For solutions to **Extra Help**, turn to **Appendix A, Topic 1**.



Extensions

You can determine the perimeter of a regular polygon without using the side length directly. If you know the number of sides and the radius of the regular polygon, you can determine its perimeter.



For a regular polygon,

$$P = \left(\sin \frac{180^\circ}{n} \right) 2nr,$$

where n is the number of sides, and r is the radius.

Look at the example. It shows how to use the calculator in evaluating P .

Example 12

What is the perimeter of a 15-sided regular polygon having a radius of 6.50 cm? Round your answer to one decimal place.

Solution:

$$\begin{aligned} P &= \left(\sin \frac{180^\circ}{n} \right) 2nr \\ &= \left(\sin \frac{180^\circ}{15} \right) (2)(15)(6.50 \text{ cm}) \end{aligned}$$

Display

0

180

180

15

12

0.20791169

0.20791169

2

0.415823381

Enter

180

15

2

Enter

15

6.50

Display

15

6.237350724

6.5

40.54277971

$P = 40.5 \text{ cm}$

The perimeter of the polygon is 40.5 cm.

Try the following questions.

1. What is the perimeter of a regular nanogon having a radius of 8.00 cm?
2. What is the perimeter of a regular heptagon having a radius of 10.0 cm?
3. A regular octagon has a radius of 7.50 m. What is the length of each side?



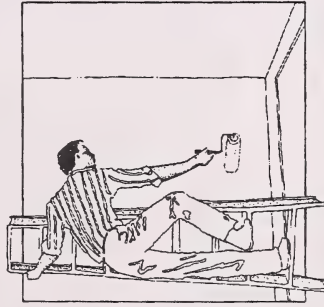
For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

Topic 2 Area



Introduction

When do you use the measurement of area? You use area, for example, when you estimate the cost of sodding a lawn, when seeding a field, and when painting a room.



You may view the video titled *Mathways*¹, program 3, as an introduction to this topic.

¹ *Mathways* is a title of Wisconsin ETV Network.



What Lies Ahead

Throughout this topic you will learn to

1. determine the area of a polygon
2. develop and use a formula for the area of a regular polygon
3. solve problems involving the area of circles
4. calculate the surface area of solids

Now that you know what to expect, turn the page to begin your study of area.



Exploring Topic 2

Activity 1

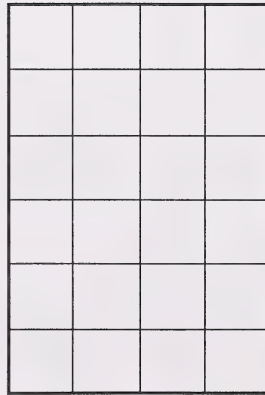
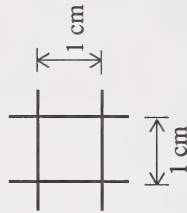


Determine the area of a polygon.

Look at the two figures. Which figure is bigger? The figure on the right is taller, but the figure on the left is wider. It is hard to make a size comparison on the basis of one dimension. You could compare the number of square units contained in each. When you do that, you find that each contains 24 square units. On the basis of this, you can say that the figures are the same size. You have used area to make this comparison.

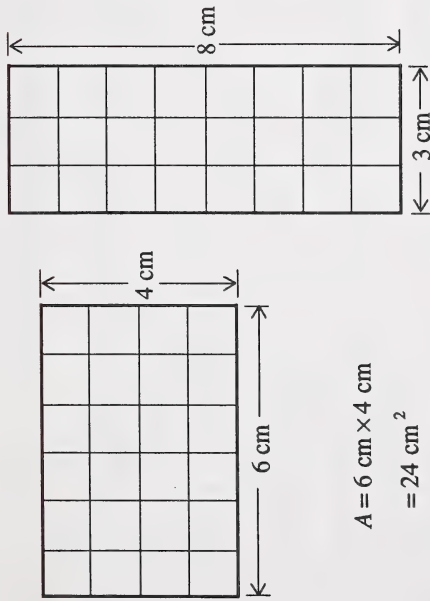
Area is a measure of the number of square units which can be enclosed in a figure.

The two rectangles discussed each enclose 24 square units. Measure the dimensions of each of these units and you will find that the sides are 1 cm long.



Such a unit is one square centimetre. This unit is 1 cm^2 . The area of each rectangle is 24 cm^2 .

The area of these rectangles can be determined by multiplying the length by the width.



$$A = 6 \text{ cm} \times 4 \text{ cm} \\ = 24 \text{ cm}^2$$

The formula for the area of a rectangle is

$$A = lw \\ (A = l \times w)$$

The unit of the length and width determines the size of the squares that you are counting for area. For example, when you use metres for the dimensions, then the area is in square metres (m^2). When kilometres are used, then the area is in square kilometres (km^2).

$$A = 3 \text{ cm} \times 8 \text{ cm} \\ = 24 \text{ cm}^2$$

Solution:

$$A = l \times w \\ = 13 \text{ cm} \times 5 \text{ cm} \\ = 65 \text{ cm}^2$$

The area of the rectangle is 65 cm^2 .

Example 2

The cost of sod is $\$1.45/\text{m}^2$ delivered. What would be the cost of sod for a rectangular lawn having a length of 50.0 m and a width of 24.5 m?

Solution:

$$A = lw \\ = 50.0 \text{ m} \times 24.5 \text{ m} \\ = 1225 \text{ m}^2$$

The cost of sod would be $1225 \text{ m}^2 \times \$1.45/\text{m}^2 = \$1776.25$.

Try the following questions. Do at least two of the following three questions.

1. What is the area of a rectangle measuring 1.5 m by 0.50 m? Express your answer in cm^2 .
2. How many square centimetres are there in a square metre?

Example 1

What is the area of the rectangle?



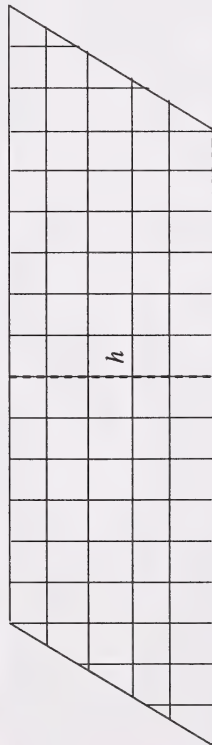
3. Margaret plans to repaint a living room wall having a width of 8.4 m and a height of 3.2 m. The paint she plans to use is sold in one-litre cans which cost \$9.45 per can. Each litre can be expected to cover 10 m^2 . How much will the paint cost Margaret?

Remember that you must round upwards for the number of cans.

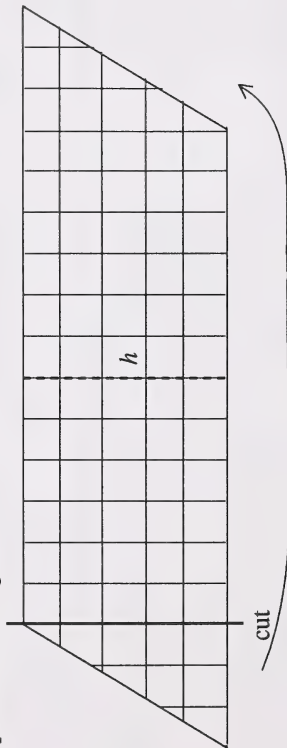


For solutions to Activity 1,
turn to Appendix A, Topic 2.

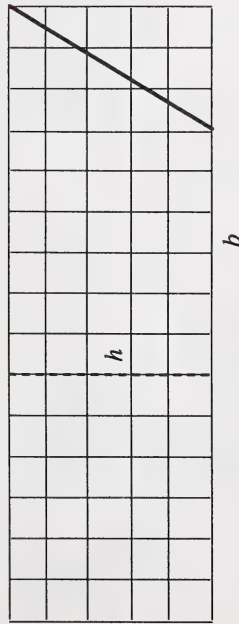
Look at the parallelogram. How can you use your knowledge of rectangles to determine the area of a parallelogram?



For the parallelogram, the height is h and the length of the base is b . A triangle can be cut off the left and placed on the right.



When that is done, you have a rectangle of dimensions b and h . For this rectangle $A = bh$.

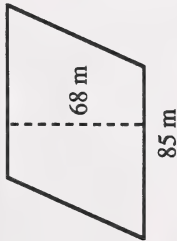


Since the area of the parallelogram and rectangle is equal, you can conclude that for a parallelogram, $A = bh$.

You can now determine the area of parallelograms directly from the base and height.

Example 3

What is the area of the parallelogram?



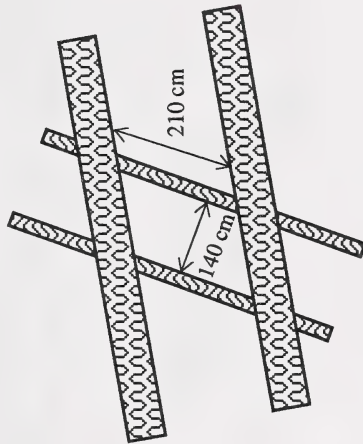
Solution:

$$\begin{aligned} A &= bh \\ &= 85 \text{ m} \times 68 \text{ m} \\ &= 5780 \text{ m}^2 \\ &\doteq 5800 \text{ m}^2 \end{aligned}$$

The area of the parallelogram is 5800 m^2 .

Example 4

What is the area of the parallelogram formed by the intersection of the tracks of two vehicles? Use the dimensions shown.



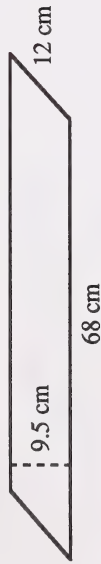
Solution:

$$\begin{aligned} A &= bh \\ &= 140 \text{ cm} \times 210 \text{ cm} \\ &= 29\,400 \text{ cm}^2 \end{aligned}$$

The area of the parallelogram is $29\,400 \text{ cm}^2$.

Try the following questions. Do at least two of the following three questions.

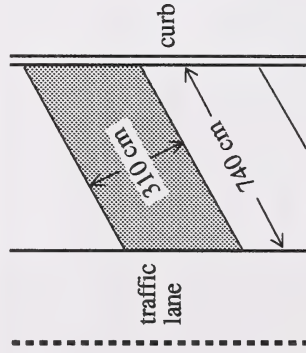
4. Calculate the area of the parallelogram.



5. Calculate the area of the parallelogram.



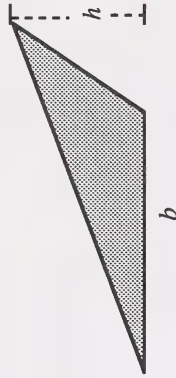
6. A parking stall is to be painted completely to indicate that it is reserved. How much area will have to be painted?



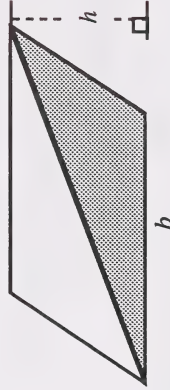
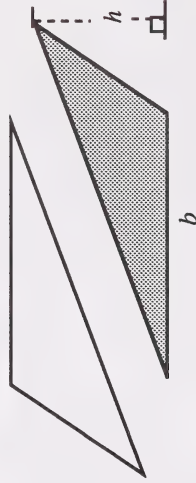
For solutions to Activity 1, turn to Appendix A, Topic 2.



What about the area of a triangle? If you place a copy of a



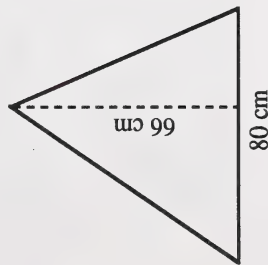
triangle on or above itself, you have a parallelogram.



The area of the resulting parallelogram is bh . You can see that the area of the triangle is half the area of the parallelogram. Therefore, for a triangle, $A = \frac{1}{2}bh$.

Example 5

Determine the area of the triangle.



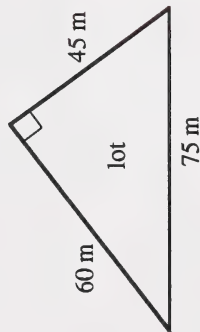
Solution:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(80 \text{ cm})(66 \text{ cm}) \\ &= 2640 \text{ cm}^2 \end{aligned}$$

The area of the triangle is 2640 cm^2 .

Example 6

Mr. and Mrs. Weslowsky are interested in a residential lot in a new development. Lots are assessed at $\$9.00 / \text{m}^2$. What would the triangular lot cost them?



Solution:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(60 \text{ m})(45 \text{ m}) \\ &= 1350 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Cost} &= \$9.00 / \text{m}^2 \times 1350 \text{ m}^2 \\ &= \$12\,150 \end{aligned}$$

The lot would cost the Weslowskys \$12 150.

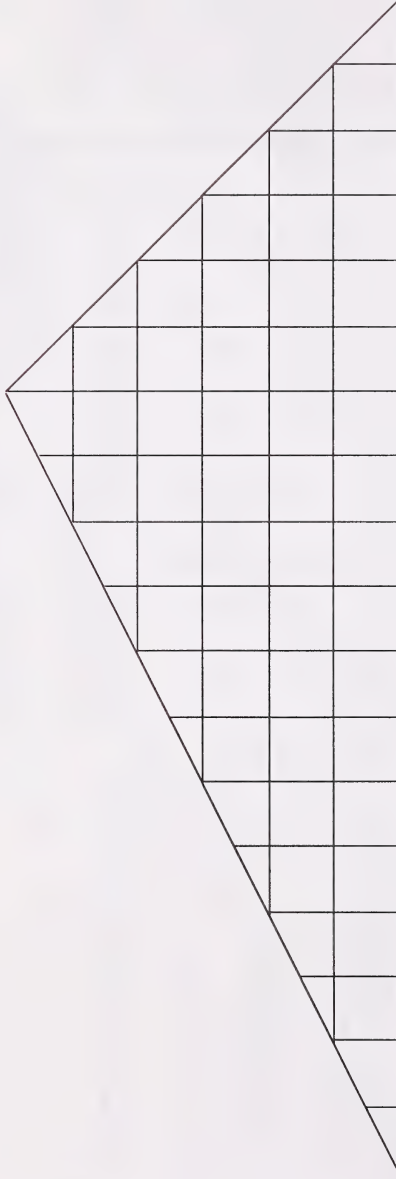
Or

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(45 \text{ m})(60 \text{ m}) \\ &= 1350 \text{ m}^2 \end{aligned}$$

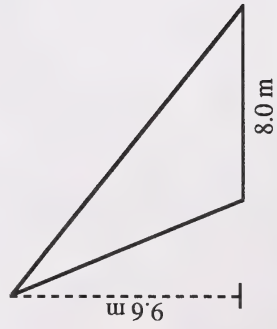
In a right triangle, either leg can serve as the height.

Try at least two of the following three questions.

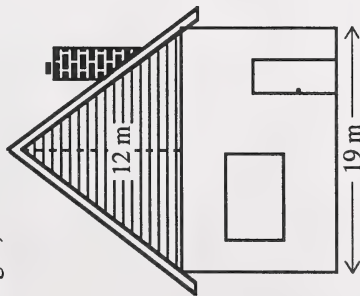
7. Find the area of the triangle. Assume each square is 1 cm^2 .



8. Find the area of the triangle.

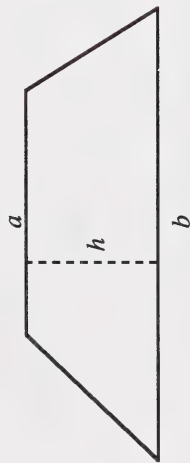


9. The Marxheimers' house has wood siding for the front gable. In order to purchase enough stain for the wood, the Marxheimers need to calculate the area of the gable. Calculate the area of the gable for the Marxheimers. (The gable is the shaded triangle.)



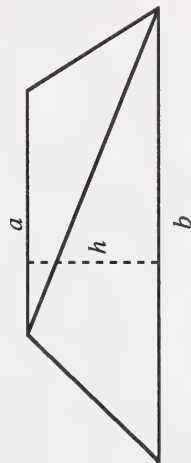
For solutions to Activity 1,
turn to **Appendix A, Topic 2**.

There is still another polygon for which the area can be found by a formula. This figure is the trapezoid.

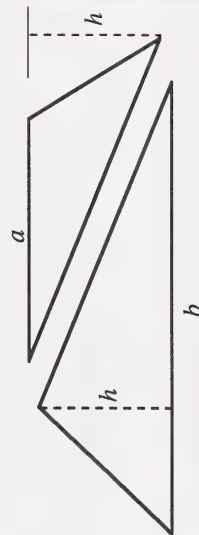


Recall that a trapezoid is a quadrilateral in which two sides are parallel.

This seems like an odd-shaped figure for which to find the area. Notice that it can be divided into two triangles.



There are two triangles, both with the same height h .



The left triangle has an area of $\frac{1}{2}bh$ and the area of the other is $\frac{1}{2}ah$. Now you can see that the area of the trapezoid must be

$$\begin{aligned} \frac{1}{2}ah + \frac{1}{2}bh \\ = \frac{1}{2}h(a+b). \end{aligned}$$

You can conclude that for a trapezoid, the formula for area is

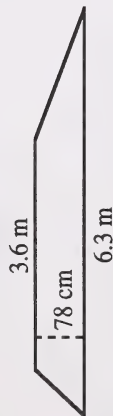
$$A = \frac{1}{2}h(a+b),$$

where a and b are the lengths of the parallel sides, and h is the distance between the parallel sides.

Look at the next example.

Example 7

What is the area of the trapezoid?

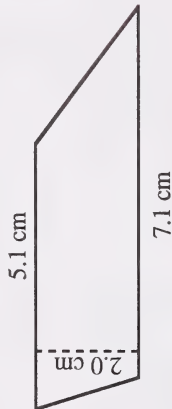


Solution:

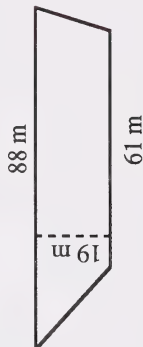
$$\begin{aligned} h &= 78 \text{ cm} \\ &= 0.78 \text{ m} \\ A &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2}(0.78 \text{ m})(3.6 \text{ m} + 6.3 \text{ m}) \\ &= 3.861 \text{ m}^2 \\ &\approx 3.9 \text{ m}^2 \text{ (rounded to one decimal place)} \end{aligned}$$

Try at least two of the following questions.

10. What is the area of the trapezoid?

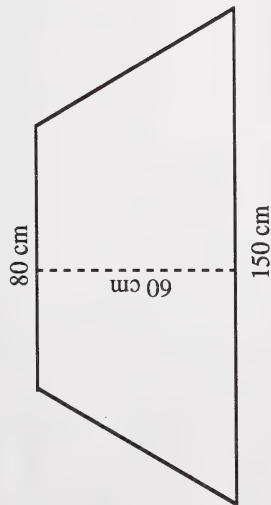


11. The figure shows the shape and dimensions of a garden. Calculate the area of the garden.



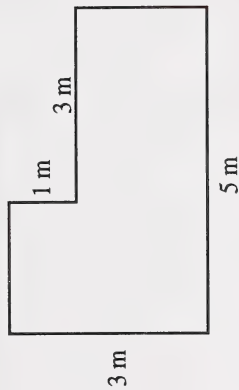
The common factor is taken out.

12. The top of a laboratory table has the shape and size shown.
What is the area of the table?

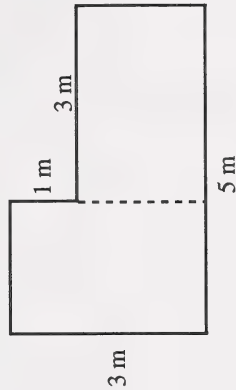


For solutions to Activity 1,
turn to **Appendix A, Topic 2.**

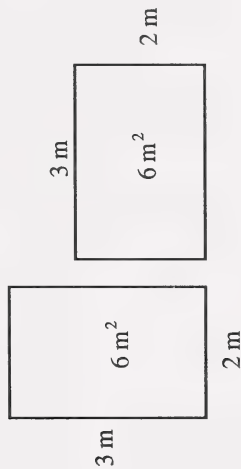
Look at the figure. You do not have a formula for its area.



You can divide the polygon into two rectangles for which you can calculate the area.



This allows you to determine the area as the sum of two rectangles.

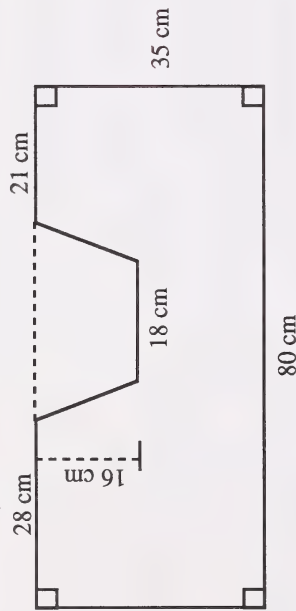


The area of the original polygon is 12 m^2 .

The next examples show how to use your area formulas for more complex polygons.

Example 8

What is the area of this octagon?



Solution:

You can think of this as a rectangle with a trapezoid removed.

The area of the rectangle is $lw = 80 \text{ cm} \times 35 \text{ cm} = 2800 \text{ cm}^2$.

The area of the trapezoid is $\frac{1}{2}h(a+b) = \frac{1}{2}(16 \text{ cm})(18 \text{ cm} + 31 \text{ cm}) = 392 \text{ cm}^2$.

The area of the octagon is $2800 \text{ cm}^2 - 392 \text{ cm}^2 = 2408 \text{ cm}^2$.

The length of the top side
of the trapezoid is

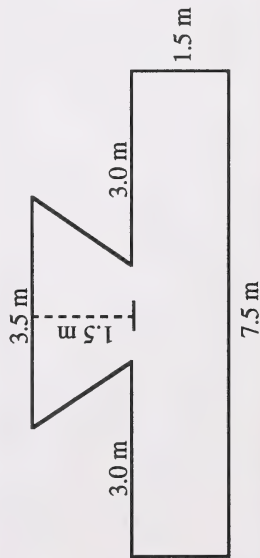
$$80 \text{ cm} - (28 \text{ cm} + 21 \text{ cm})$$

$$= 80 \text{ cm} - 49 \text{ cm}$$

$$= 31 \text{ cm}.$$

Example 9

What is the area of the garden represented by the figure?



Solution:

The area can be divided into a trapezoid and a rectangle. The length of the bottom side of the trapezoid is $7.5 \text{ m} - (3.0 \text{ m} + 3.0 \text{ m}) = 1.5 \text{ m}$. For the trapezoid,

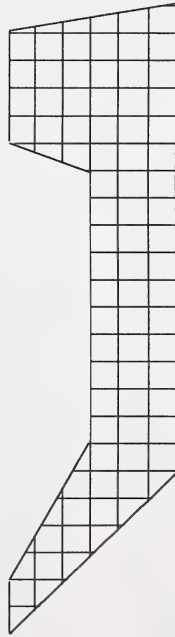
$$\begin{aligned} A &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2}(1.5 \text{ m})(1.5 \text{ m} + 3.5 \text{ m}) \\ &= \frac{1}{2}(1.5 \text{ m})(5.0 \text{ m}) \\ &= 3.75 \text{ m}^2. \end{aligned}$$

$$\begin{aligned} \text{Area of the rectangle} &= 1.5 \text{ m} \times 7.5 \text{ m} \\ &= 11.25 \text{ m}^2 \end{aligned}$$

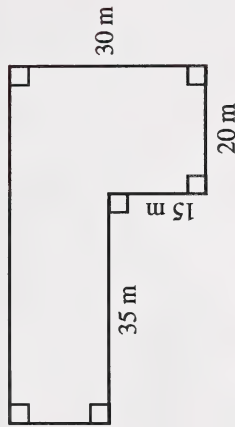
$$\begin{aligned} \text{Total area of the garden} &= 3.75 \text{ m}^2 + 11.25 \text{ m}^2 \\ &= 15.0 \text{ m}^2 \end{aligned}$$

Try the following questions.

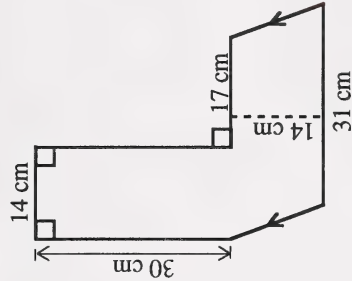
13. What is the area covered by the grid? Assume each grid square represents 1 m^2 .



14. What is the area of the figure?



15. Suppose you wanted to cut out of cardboard a stylized L as shown by the figure. How many square centimetres of cardboard would make up the letter?



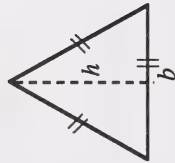
For solutions to Activity 1, turn to Appendix A, Topic 2.

Activity 2

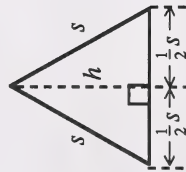


Develop and use a formula for the area of a regular polygon.

A regular polygon, as you recall, is one whose sides and angles are the same. The simplest regular polygon is the equilateral triangle. To find the area of such a triangle, use the formula $A = \frac{1}{2}bh$ which applies to any triangle.



For an equilateral triangle, the area formula can be simplified because the value of h can be found from the side length, s .



Referring to the diagram, you can see that the theorem of Pythagoras can be applied.

$$h^2 + \left(\frac{1}{2}s\right)^2 = s^2$$

$$h^2 + \frac{1}{4}s^2 = s^2$$

$$h^2 = s^2 - \frac{1}{4}s^2$$

$$= \left(1 - \frac{1}{4}\right)s^2$$

$$= \frac{3}{4}s^2$$

$$h = \sqrt{\frac{3}{4}s}$$

$$= \frac{\sqrt{3}}{2}s$$

You can substitute s for b and $\frac{\sqrt{3}}{2}s$ for h in the formula $A = \frac{1}{2}bh$.

$$A = \frac{1}{2}s\left(\frac{\sqrt{3}}{2}s\right)$$

$$= \frac{\sqrt{3}}{4}s^2$$

It follows that for an equilateral triangle with side length of s ,

$$A = \frac{\sqrt{3}}{4}s^2.$$

s^2 is factored out.

Example 10

Calculate the area of the equilateral triangle having a perimeter of 18 cm.

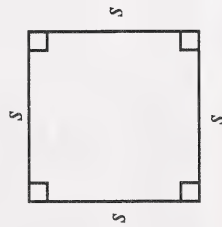
Solution:

Each of the sides measures $18 \text{ cm} \div 3 = 6.0 \text{ cm}$.

$$\begin{aligned} A &= \frac{\sqrt{3}}{4} s^2 \\ &= \frac{\sqrt{3}}{4} (6.0 \text{ cm})^2 \\ &= \frac{1.73}{4} (36 \text{ cm}^2) \\ &= 15.57 \text{ cm}^2 \\ &\approx 16 \text{ cm}^2 \quad (\text{rounded to the nearest square centimetre}) \end{aligned}$$

For this triangle, the area is 16 cm^2 .

The square is a quadrilateral with its four sides the same length. For this figure, the area formula of a rectangle can be applied.



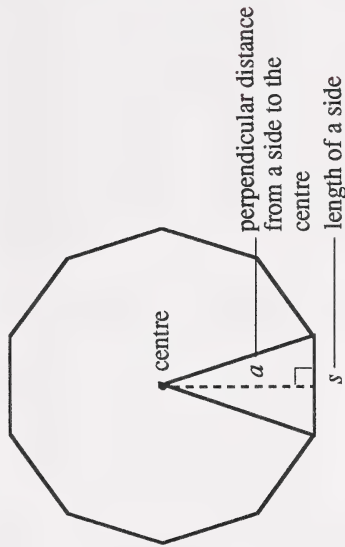
For each of l and w , the side length, s , can be substituted to obtain a simpler formula.

$$\begin{aligned} A &= lw \\ &= s \times s \\ &= s^2 \end{aligned}$$

Therefore, for a square

$$A = s^2.$$

It is possible to develop an area formula that applies to all regular polygons and for polygons with more than four sides. Such a general formula is very useful. You can develop such a formula for regular polygons by studying a decagon. In the next questions, you will find the area of a regular polygon in terms of distance from the centre to a side, a , and the side length, s .



The sides of a polygon meet at points called **vertices**. The vertices of a regular polygon all lie on a circle. The centre of this circle is said to be the **centre** of the polygon.

Try the following questions. Refer to the decagon.

1. What is the area of the triangle drawn in the decagon? Express your answer in terms of a and s .
2. How many triangles, like the one drawn, fit in the decagon? You may want to join the centre to each vertex to help your counting.
3. How much larger is the area of the decagon as compared to the triangle?
4. Give the area of the complete decagon in terms of a and s .
5. Generalize the area formula for a decagon so that it applies to any n -sided regular polygon.

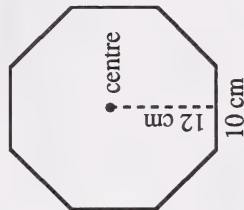


For solutions to **Activity 2**, turn to **Appendix A, Topic 2**.

In your work you have been able to find that for a regular polygon with n sides, $A = \frac{1}{2}ans$, where a is the perpendicular distance from a side to the centre of the polygon, n is the number of sides of the polygon, and s is the side length of each of the sides of the polygon.

Example 11

What is the area of the regular octagon?



Hint: Substitute s for b and a for h in the formula
 $A = \frac{1}{2}bh$.

Solution:

$$\begin{aligned} A &= \frac{1}{2}ans \\ &= \frac{1}{2}(12 \text{ cm})(8)(10 \text{ cm}) \\ &= 480 \text{ cm}^2 \end{aligned}$$

The area of the regular octagon is 480 cm^2 .

Use n in your formula to stand for the number of sides.

Example 12

A regular 12-sided polygon has a perimeter of 26.9 m. The distance from a side to the centre is 3.90 m. Calculate the area of the polygon.

Solution:

$$s = \frac{26.9 \text{ m}}{12}$$

$$= 2.24 \text{ m}$$

$$A = \frac{1}{2} \text{ ans}$$

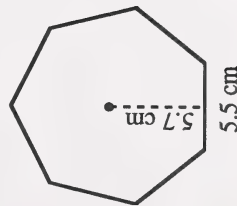
$$= \frac{1}{2} (3.90 \text{ m})(12)(2.24 \text{ m})$$

$$= 52.416 \text{ m}^2$$

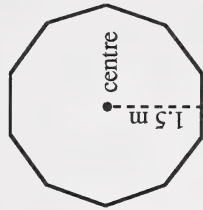
$$\approx 52.4 \text{ m}^2$$

Try the following questions.

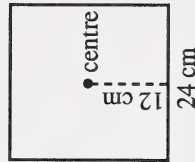
6. Find the area of the regular polygon.



7. Find the area of the regular polygon. Its perimeter is 9.7 m.



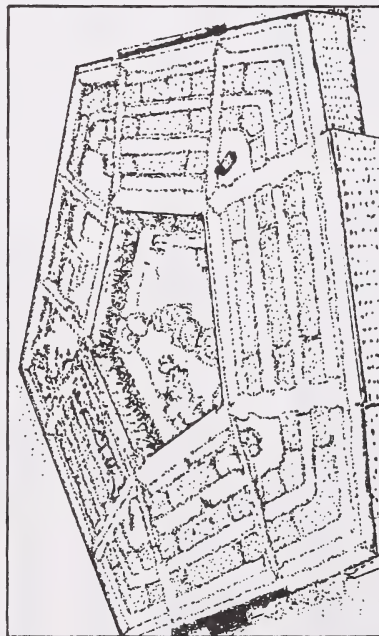
8. Find the area of the square.



9. What is the area of an equilateral triangle having a perimeter of 42 m?
10. What is the area of the flat roof of a house which is octagonal in shape? The distance between parallel sides of the roof is 18 m. Each of the eight sides of the roof measures 10 m.
11. What is the area of an equilateral triangle in which each side measures 12 cm?

12. A sixteen-sided regular polygon has a side length of 24 cm. The distance from the centre to the sides of the polygon is 60 cm. What is the area of the polygon?

13. The Pentagon Building is one of the largest office buildings in the world. It is the headquarters for the U.S. Department of Defence. Its outermost walls are close to 1.6 km around. This building is in the shape of a regular pentagon. The distance from the pentagon centre to each wall is 220 m. Calculate the area occupied by the Pentagon building.



14. What is the length of each side of a square having an area of 45 cm^2 ? Round your answer to one decimal place.



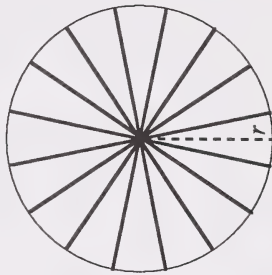
For solutions to Activity 2, turn to Appendix A, Topic 2.

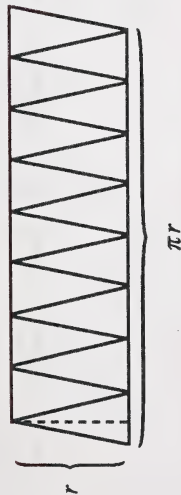
Activity 3



Solve problems involving the area of circles.

The circle can be converted to a figure of the same area by cutting the circle into many pie-shaped pieces and putting them together.





The new figure is very close to a parallelogram with a height of r and a base length of πr . For such a figure, $A = bh = (\pi r)r = \pi r^2$. Therefore, you can conclude that for circles, $A = \pi r^2$.

Example 13

What is the area of a circle with a radius of 3.0 cm?

Solution:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi (3.0 \text{ cm})^2 \\ &= \pi (9.0 \text{ cm}^2) \\ &\doteq 28.3 \text{ cm}^2 \end{aligned}$$

The area of the circle is 28.3 cm^2 .

Example 14

Pizza sizes are often given in terms of diameter. If a 25 cm pizza costs \$8.95, what is the cost per cm^2 of pizza?

Solution:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{25 \text{ cm}}{2} \right)^2 \\ &= \pi (12.5 \text{ cm})^2 \\ &= (\pi) (156.25 \text{ cm}^2) \\ &\doteq 491 \text{ cm}^2 \text{ (rounded to the nearest square centimetre)} \end{aligned}$$

$$\begin{aligned} \text{Cost per cm}^2 &= \frac{\$8.95}{491 \text{ cm}^2} \\ &= \$0.018 / \text{cm}^2 \\ &= 1.8\phi / \text{cm}^2 \text{ (rounded to the nearest tenth of a cent)} \end{aligned}$$

The cost of the pizza is $1.8\phi / \text{cm}^2$.

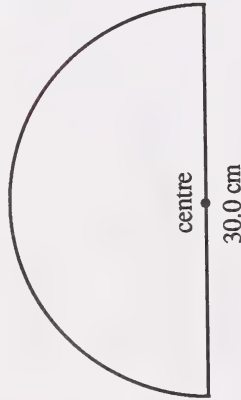
Half of the pieces have their rounded edge along the bottom. Therefore, the base is equal to half the circumference of the circle.

$$\begin{aligned} C &= 2\pi r \text{ and} \\ \frac{1}{2} C &= \frac{2\pi r}{2} \pi \end{aligned}$$

$$\begin{aligned} \text{Remember: } d &= 2r \\ \therefore r &= \frac{d}{2} \end{aligned}$$

Example 15

What is the area of this semicircle?



Solution:

The figure has half the area of a circle of radius 15.0 cm. For the complete circle,

$$\begin{aligned} A &= \pi r^2 \\ &= \pi (15.0 \text{ cm})^2 \\ &\doteq 706.9 \text{ cm}^2 \end{aligned}$$

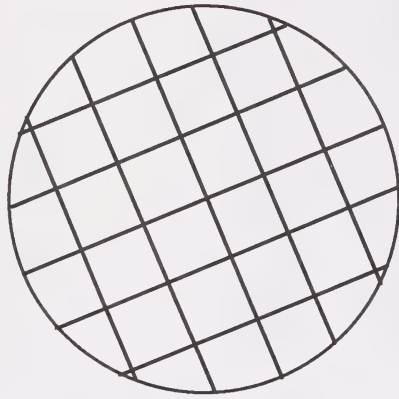
For the semicircle,

$$\begin{aligned} A &= \frac{1}{2} (706.9 \text{ cm}^2) \\ &\doteq 353 \text{ cm}^2 \quad (\text{to the nearest square centimetre}) \end{aligned}$$

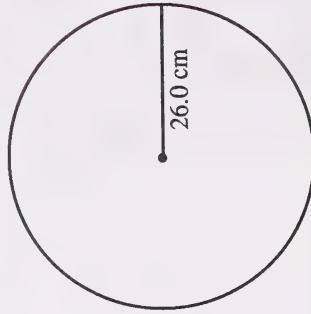
The area of the semicircle is 353 cm^2 .

Try these questions. Do at least five of the next eight questions.

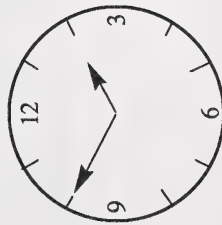
1. What is the area of the circle? Each square represents 1 cm^2 .



2. What is the area of the circle?



3. What is the area of a circle with diameter 43.6 cm?
4. What is the area of a circle with radius 1.00 m?
5. What is the area of a circle with radius 17.8 cm?
6. What is the area of the face of the watch? It is drawn life-size.



7. A pie plate measures 22.5 cm across. What area of dough is needed to make the top crust?
8. At Princely Pizzas the 18 cm Hawaiian pizzas are priced at \$5.49. The 26 cm Hawaiian pizza costs \$8.25. Which is the better buy? Give reasons for your answer.

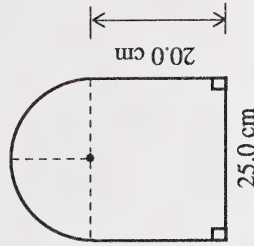


For solutions to Activity 3, turn to Appendix A, Topic 2.

Some figures are partly circular and partly polygonal. The area is then calculated by dividing the given figure into manageable figures.

Example 16

Find the area of the figure. The circular part forms a semicircle.

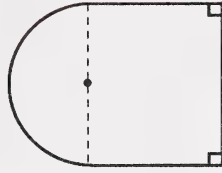


Solution:

The area can be divided into a semicircle and a rectangle.

For the semicircle,

$$\begin{aligned}
 A &= \frac{1}{2} \pi r^2 \\
 &= \frac{1}{2} (\pi) \left(\frac{25.0 \text{ cm}}{2} \right)^2 \\
 &= \frac{1}{2} (\pi) (12.5 \text{ cm})^2 \\
 &\approx 245 \text{ cm}^2 \quad (\text{rounded to the nearest square centimetre})
 \end{aligned}$$



For the rectangle,

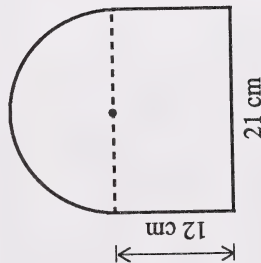
$$\begin{aligned} A &= lw \\ &= (25.0 \text{ cm})(20.0 \text{ cm}) \\ &= 500 \text{ cm}^2 \end{aligned}$$

The total area is $245 \text{ cm}^2 + 500 \text{ cm}^2 = 745 \text{ cm}^2$.

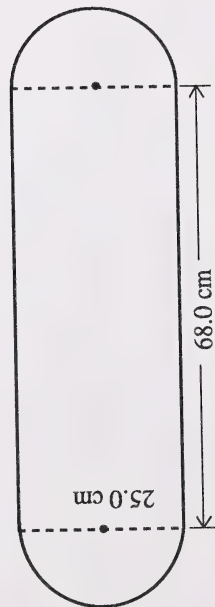
The area of the complete figure is 745 cm^2 .

Try the following questions.

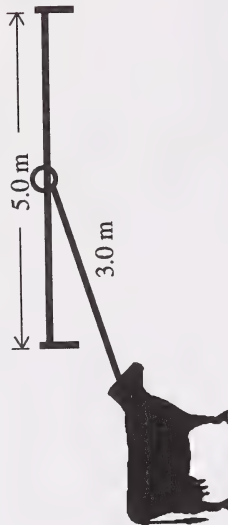
9. What is the area of the figure?



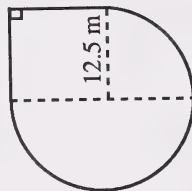
10. What is the area of the figure?



11. Bessie the cow is attached to a ring by a 3 m rope. The ring can slide from one end to the other end of the 5 m line. What area of grass is accessible to Bessie?



12. What is the area of the figure?



For solutions to Activity 3, turn to Appendix A, Topic 2.

Suppose you were given a permit to settle on crown land. Rather than specify an area of land you were entitled to, the permit allowed you a choice of property as long as the perimeter of the land was 4000 m. What shape would give you the most land? Would your property be rectangular, square, circular, or even some other shape? Such a problem is one of finding the largest area for a given perimeter. The next four questions deal with this.

Try the following questions.

13. Use the chart which follows to record your work for parts a and b.
 - a. Determine the perimeter of the following figures.
 - b. Determine the area of the following figures.
 - c. Which figure has the largest area?



Figure A

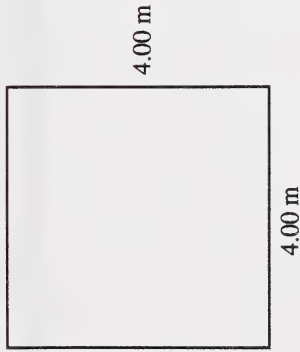


Figure B

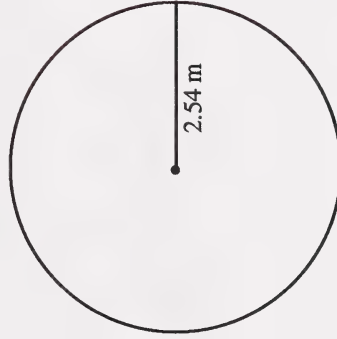


Figure C



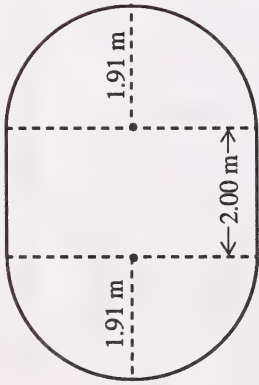


Figure D

	Figure A	Figure B	Figure C	Figure D
a. Perimeter				
b. Area				

14. Use the chart which follows to record your work for parts a and b.

- Determine the perimeter of the following figures.
- Determine the area of the following figures.
- Which figure has the largest area?

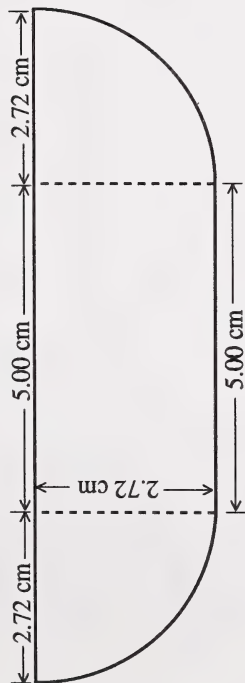


Figure A

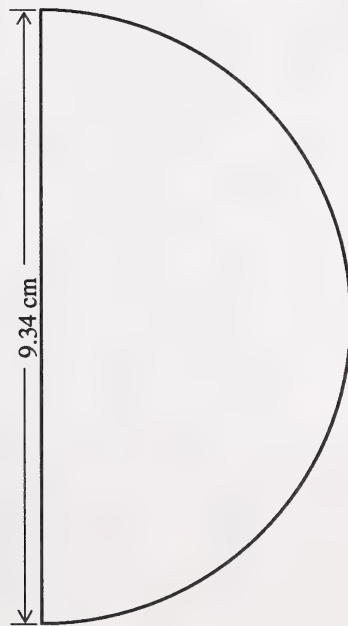


Figure B

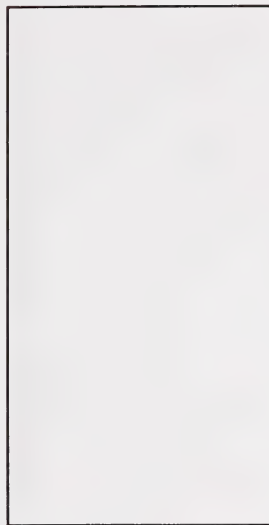


Figure C

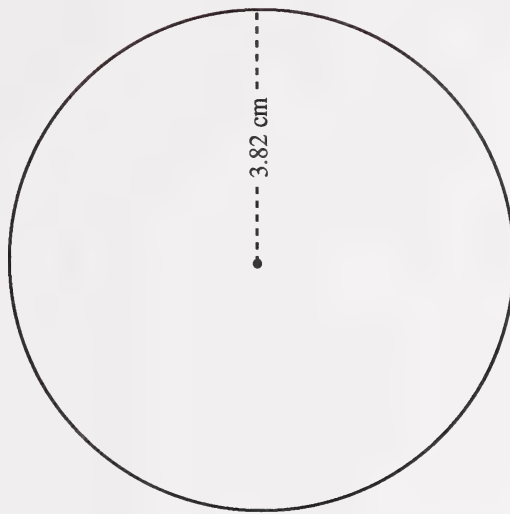


Figure D

	a. Perimeter	b. Area
Figure A		
Figure B		
Figure C		
Figure D		

15. Each of the following has a perimeter of close to 4000 m.

- a. Which figure encloses the most area?
- b. What shape would you make your land if your permit allowed you a property with a perimeter of 4000 m?

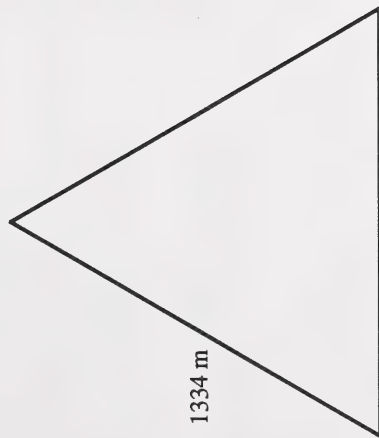


Figure A

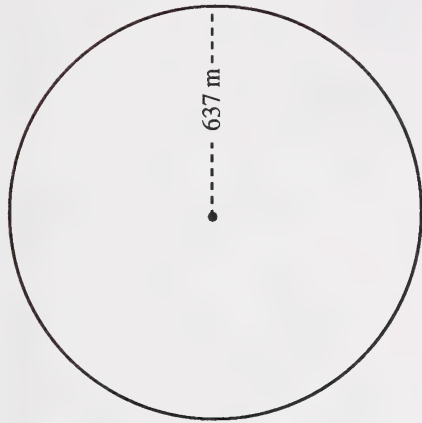


Figure B

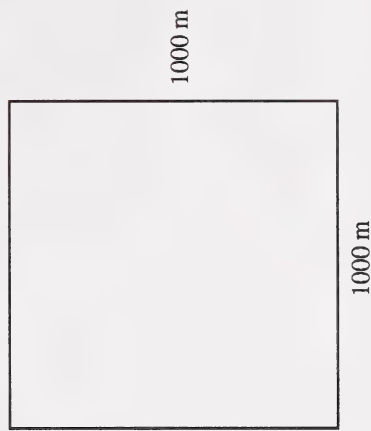
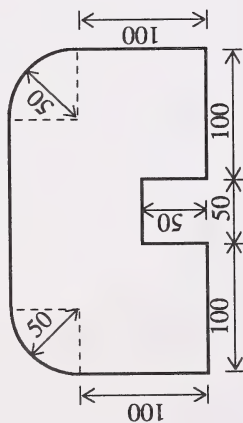


Figure C

16. Based on your experience, what figure encloses the greatest area for a given perimeter?

17. The diagram shows the floor plan of a public building. Determine the area that the building occupies.



* All dimensions are in metres.



For solutions to Activity 3, turn to Appendix A, Topic 2.

Activity 4

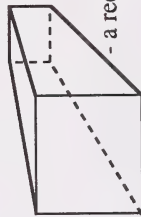


Calculate the surface area of solids.

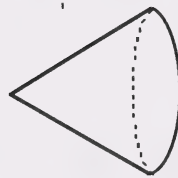
In this unit, you have studied figures that have only two dimensions and exist in a plane. Now you will study three-dimensional figures that take up space. Such figures are called solids. The following figures are solids.



- a cylinder



- a rectangular prism



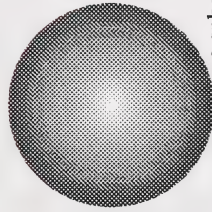
- a cone



- a pyramid



- a triangular prism



- a sphere

In order to determine the surface area of a solid, it is helpful to represent its surface in a two-dimensional manner. Such a representation is called a net of a solid. The following questions use familiar figures to help you develop a connection between a solid and its net.

For nets for Activity 4, turn to **Appendix B**, nets 1 through 5.

Try the following question using the nets.

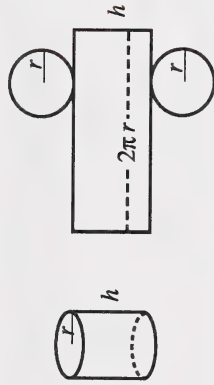
1. Make the following figures by cutting out the nets in **Appendix B**.

- a. cylinder
- b. cone
- c. pyramid
- d. rectangular prism
- e. triangular prism



For solutions to Activity 4, turn to **Appendix A, Topic 2**.

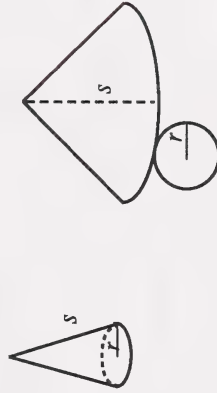
The surface area of a solid is equal to the area of its net.



For the cylinder, the net shows that the surface area is composed of two circular parts and a rectangle. The area of each of the circles is πr^2 . The length of the rectangle is the circumference of the circle. The area of the rectangle is the product of its length and width which is $(2\pi r)h$.

You can conclude that for the cylinder,

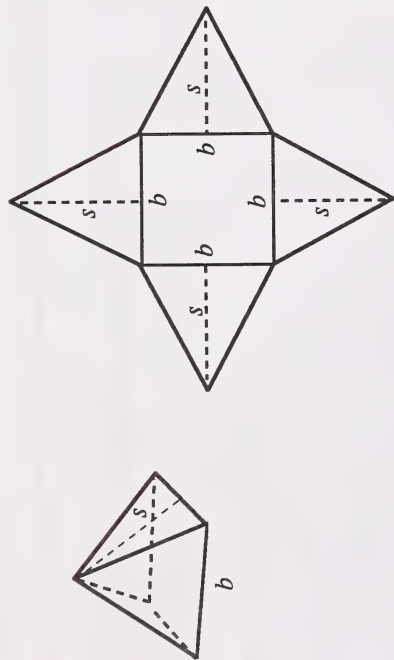
$$A = 2\pi rh + 2\pi r^2.$$



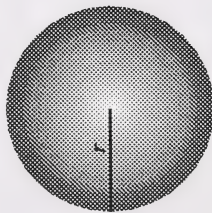
The surface area of the cone is the sum of the areas of a circle and a sector of a circle. The area of the circular base is πr^2 . The area of the sector is $(\pi r)s$. For a cone, $A = \pi r^2 + \pi rs$.

In this activity you will make a number of solids from nets. Keep your models for other activities in this unit.

Fold on dotted lines of the net. Flaps are included to make attachments. You may use glue or tape.



The surface area of the pyramid is the sum of the areas of four triangular sides and a square base. The area of each of the triangles is $\frac{1}{2}bs$. The area of four such triangles is $4(\frac{1}{2}bs) = 2bs$. The area of the square base is b^2 . For the pyramid, $A = 2bs + b^2$. It is important to remember that s is the shortest distance from the base to the top along the side. This is the slant height. It is not the perpendicular distance from the base to the top.

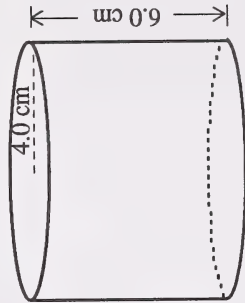


The sphere cannot be represented by a net, so it is not easy to give reasons for the surface area formula. This is accepted for a sphere: $A = 4\pi r^2$. The surface area of a sphere is four times the area of a circle of the same radius.

A prism has congruent polygons as ends. These ends are parallel. Its sides are rectangles. You have made a rectangular prism and a triangular prism. The prism is named by the shape of its ends. To find the surface area of a prism, add up the areas of the ends and sides. Drawing a net may help you.

Example 17

What is the surface area of the soup can?



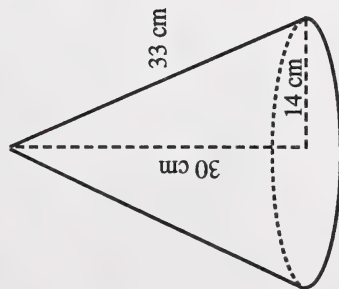
Solution:

$$\begin{aligned}
 A &= 2\pi rh + 2\pi r^2 \\
 &= 2\pi(4.0 \text{ cm})(6.0 \text{ cm}) + 2\pi(4.0 \text{ cm})^2 \\
 &= 2\pi[(4.0 \text{ cm})(6.0 \text{ cm}) + (4.0 \text{ cm})^2] \\
 &= 2\pi(24.0 \text{ cm}^2 + 16.0 \text{ cm}^2) \\
 &= 2(\pi)(40.0 \text{ cm}^2) \\
 &\approx 251 \text{ cm}^2
 \end{aligned}$$

The surface area of the soup can is 251 cm^2 .

Example 18

What is the surface area of the cone?



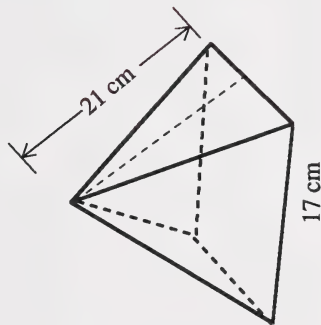
Solution:

$$\begin{aligned} A &= \pi r^2 + \pi rs \\ &= (\pi)(14 \text{ cm})^2 + (\pi)(14 \text{ cm})(33 \text{ cm}) \\ &\doteq 616 \text{ cm}^2 + 1451 \text{ cm}^2 \\ &\doteq 2067 \text{ cm}^2 \end{aligned}$$

The surface area of the cone is 2067 cm^2 .

Example 19

What is the surface area of the pyramid?



Solution:

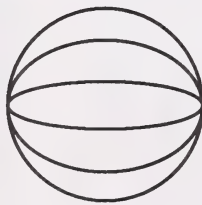
$$\begin{aligned} A &= 2bs + b^2 \\ &= 2(17 \text{ cm})(21 \text{ cm}) + (17 \text{ cm})^2 \\ &= 714 \text{ cm}^2 + 289 \text{ cm}^2 \\ &= 1003 \text{ cm}^2 \end{aligned}$$

The surface area of the pyramid is 1003 cm^2 .

Remember that the slant height is used.

Example 20

What is the surface area of a beach ball having a diameter of 78.0 cm?



Solution:

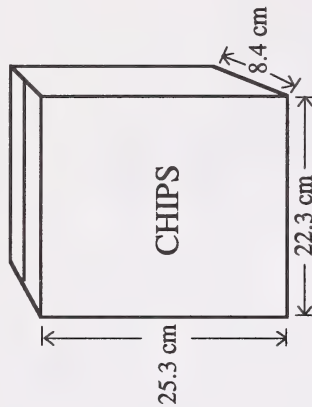
$$r = \frac{78.0 \text{ cm}}{2}$$
$$= 39.0 \text{ cm}$$

$$A = 4\pi r^2$$
$$= 4(\pi)(39.0 \text{ cm})^2$$
$$\doteq 19\,113 \text{ cm}^2$$

The surface area of the beach ball is 19 113 cm².

Example 21

How many square centimetres of cardboard are needed to complete the box?



Solution:

For each of the front and back, $A = 25.3 \text{ cm} \times 22.3 \text{ cm}$
 $= 564.19 \text{ cm}^2$.

For each of the left and right sides, $A = 25.3 \text{ cm} \times 8.4 \text{ cm}$
 $= 212.52 \text{ cm}^2$.

For each of the top and bottom, $A = 22.3 \text{ cm} \times 8.4 \text{ cm}$
 $= 187.32 \text{ cm}^2$.

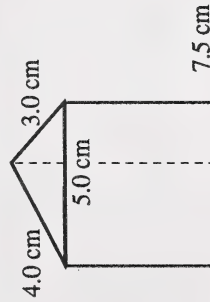
For the box,

$$A = 2(564.19 \text{ cm}^2) + 2(212.52 \text{ cm}^2) + 2(187.32 \text{ cm}^2)$$
$$\doteq 1928 \text{ cm}^2$$

There are 1928 cm² of cardboard needed to complete the box.

Example 22

What is the surface area of the prism? The ends are right triangles.



For side 3:

$$A = (5.0 \text{ cm})(7.5 \text{ cm}) \\ = 37.5 \text{ cm}^2$$

For the surface area:

$$A = 2(6.0 \text{ cm}^2) + 30 \text{ cm}^2 + 22.5 \text{ cm}^2 + 37.5 \text{ cm}^2 \\ = 102 \text{ cm}^2$$

Solution:

For the area of each of the triangular ends

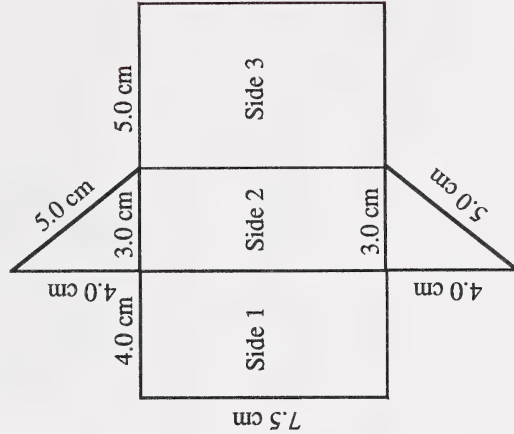
$$A = \frac{1}{2}bh \\ = \frac{1}{2}(3.0 \text{ cm})(4.0 \text{ cm}) \\ = 6.0 \text{ cm}^2$$

For side 1:

$$A = (4.0 \text{ cm})(7.5 \text{ cm}) \\ = 30 \text{ cm}^2$$

For side 2:

$$A = (3.0 \text{ cm})(7.5 \text{ cm}) \\ = 22.5 \text{ cm}^2$$



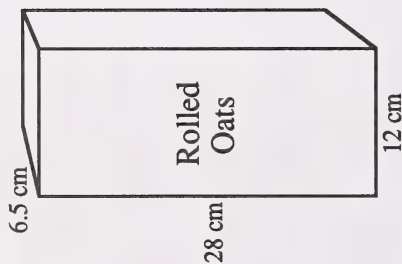
The surface area of the prism is 102 cm^2 .

Try the following questions. Do at least the even-numbered questions.

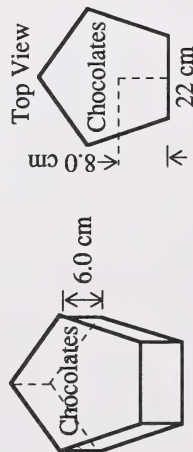
2. What is the surface area of a rectangular prism measuring

$$1.25 \text{ m} \times 4.50 \text{ m} \times 2.00 \text{ m}?$$

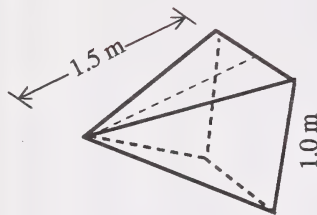
3. What is the surface area of the cereal box?



4. What is the surface area of a box of chocolates in the shape of a regular pentagonal prism?

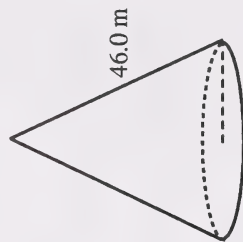


5. What is the surface area of the pyramid?



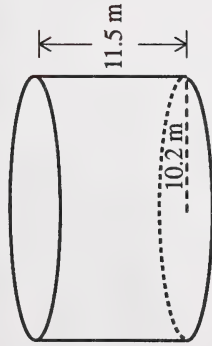
6. What is the surface area of a pyramid for which the slant height is 26 m and the base measures $15 \text{ m} \times 15 \text{ m}$?

7. What is the surface area of the cone-shaped sculpture? The radius of the base of the cone is 20 m.



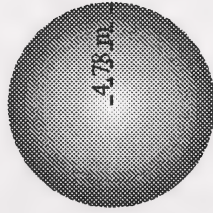
8. A cone has a slant height of 35.0 cm. The radius of its circular base is 24.0 cm. What is the surface area of the cone?

9. What is the surface area of the cylinder?



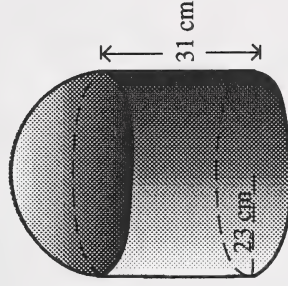
10. A can of spaghetti has a radius of 41 cm and a height of 68 cm. A label covers the curved side of the can. What is the area of the label?

11. What is the surface area of the sphere?



12. What is the surface area of a sphere of radius 8.08 cm?

13. What is the surface area of the figure? The top part forms a hemisphere. The rest is cylindrical in shape.



For solutions to Activity 4, turn to **Appendix A, Topic 2.**

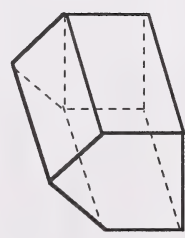
Solids with surfaces made up of flat areas only are called **polyhedrons**. The cone and sphere are not polyhedrons because their surfaces are curved or partly curved. The rectangular prism, the triangular prism, and the pyramid are examples of polyhedrons.

Each flat area of the polyhedron surface is called a face. A polyhedron can be named according to the number of faces it has.

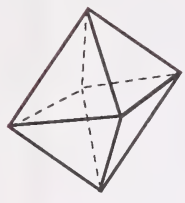
Do you recognize the same prefixes used to name polyhedrons? Each name for a polyhedron ends in "hedron".

Number of Faces	Name of Polyhedron
4	tetrahedron
5	pentahedron
6	hexahedron
7	heptahedron
8	octahedron
9	nonahedron
10	decahedron
12	dodecahedron
20	icosahedron

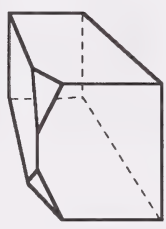
The pyramid has five faces, so it is a pentahedron. The rectangular prism has six faces, so it is a hexahedron.



The figure shown above has seven faces and can be called a heptahedron.



You can think of this figure as the result of sticking two pyramids together at their bases. The result is an octahedron. The following is also an octahedron.



It is formed by cutting two corners off a rectangular prism.

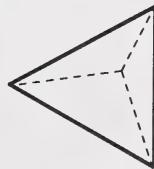
Try the following questions.

14. Draw a tetrahedron.
15. Draw two different hexahedrons.

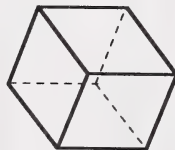


For solutions to Activity 4, turn to Appendix A, Topic 2.

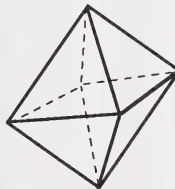
Of particular interest are regular polyhedrons. A vertex of a polyhedron is a point where several faces meet. A regular polyhedron is one which has its vertices on an imaginary sphere that encloses the polyhedron. All the faces of a regular polyhedron are regular polygons of the same size and shape. The following are the only five regular polyhedrons.



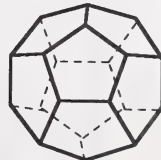
Regular
Tetrahedron



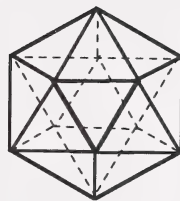
Regular
Hexahedron
(Cube)



Regular
Octahedron



Regular
Dodecahedron



Regular
Icosahedron

For nets for Activity 4, turn to **Appendix B**, nets 6 through 10.

Try the following questions using the nets. Do at least the even-numbered questions.

16. Make the following polyhedrons.
 - a. regular tetrahedron
 - b. regular hexahedron
 - c. regular octahedron
17. Make the following polyhedrons.
 - a. regular dodecahedron
 - b. regular icosahedron

This is a challenging exercise. Do not be concerned if you are not able to construct these solids.

18. For which regular polyhedrons are the faces equilateral triangles?



For solutions to Activity 4, turn to **Appendix A**, Topic 2.

The surface area of a regular polyhedron can be found by using the fact that the surface area equals the area of one face multiplied by the number of faces of the polyhedron.

$$A = A_{\text{face}} n$$

Example 23

What is the surface area of a dodecahedron in which each face has an area of 42 cm^2 ?

Solution:

$$\begin{aligned} A &= A_{\text{face}} n \\ &= (42 \text{ cm}^2)(12) \\ &= 504 \text{ cm}^2 \end{aligned}$$

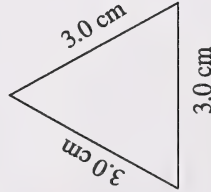
The surface area of the dodecahedron is 504 cm^2 .

Example 24

What is the surface area of an icosahedron in which the edge length is 3.0 cm ? Round your answer to the nearest square centimetre.

Solution:

The icosahedron is one of the polyhedrons having faces in the shape of equilateral triangles. The side length of such a triangle is 3.0 cm .



$$\begin{aligned} A_f &= \frac{\sqrt{3}}{4} s^2 \\ &= \left(\frac{\sqrt{3}}{4} \right) (3.0 \text{ cm})^2 \\ &= \left(\frac{\sqrt{3}}{4} \right) (9.0 \text{ cm}^2) \\ &\approx 3.897 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= A_{\text{face}} n \\ &= (3.897 \text{ cm}^2)(20) \\ &\approx 78 \text{ cm}^2 \end{aligned}$$

The surface area of the icosahedron is 78 cm^2 .

A_{face} represents the area of one face.

Edge - the line segment along which two faces of a polyhedron meet.

See Example 10 for the calculation of the area of an equilateral triangle.

Try the following questions. Do at least the odd-numbered questions.

19. What is the surface area of a regular hexahedron in which each edge is 12.0 cm long?
20. What is the surface area of a regular tetrahedron in which each edge is 15 cm long?
21. What is the surface area of a regular dodecahedron in which each face has an area of 5.8 m^2 ?
22. What is the surface area of a regular octahedron in which the edge length is 7.0 cm?
23. What is the surface area of a regular icosahedron in which each face has an area of 4.2 m^2 ?
24. The surface area of a regular octahedron is 640 cm^2 . What is the length of each edge of the octahedron?



For solutions to Activity 4, turn
to **Appendix A, Topic 2.**

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

The formulas in the table can be used to find area.

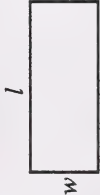
Figure	Formula for Area
	$A = \frac{1}{2}bh$
	$A = lw$
	$A = s^2$
	$A = \pi r^2$

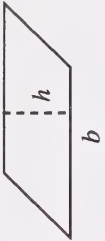
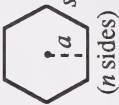
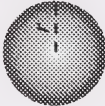
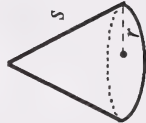
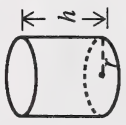
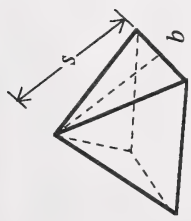
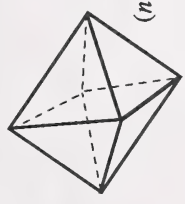
Figure	Formula for Area
	$A = bh$
	$A = \frac{1}{2}h(a+b)$
 (n sides)	$A = \frac{1}{2}ans$
	$A = 4\pi r^2$
	$A = \pi r^2 + \pi rs$

Figure	Formula for Area
 Cylinder	$A = 2\pi rh + 2\pi r^2$
 Pyramid	$A = 2bs + b^2$
 Regular Polyhedron	$A = A_{\text{face}} n$

Look at the examples.

Example 25

What is the area of a parallelogram having a height of 10.8 cm and a base measuring 4.9 cm?

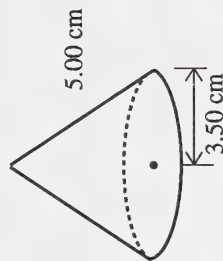
Solution:

$$\begin{aligned}
 A &= bh \\
 &= (10.8 \text{ cm})(4.9 \text{ cm}) \\
 &\doteq 53 \text{ cm}^2
 \end{aligned}$$

The area of the parallelogram is 53 cm².

Example 26

What is the surface area of the cone?



Solution:

$$\begin{aligned}
 A &= \pi r^2 + \pi rs \\
 &= (\pi)(3.5 \text{ cm})^2 + (\pi)(3.5 \text{ cm})(5.0 \text{ cm}) \\
 &\doteq 38.485 \text{ cm}^2 + 54.978 \text{ cm}^2 \\
 &\doteq 93.5 \text{ cm}^2
 \end{aligned}$$

The surface area of the cone is 93.5 cm².

Example 27

What is the surface area of a regular octahedron in which each face has an area of 3.5 m^2 ?

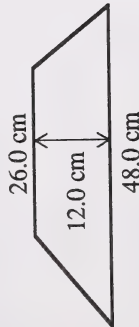
Solution:

$$\begin{aligned} A &= A_{\text{face}} n \\ &= (3.5 \text{ m}^2) 8 \\ &= 28 \text{ m}^2 \end{aligned}$$

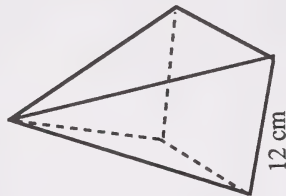
The surface area of a regular octahedron is 28 m^2 .

Try the following questions in which certain area formulas are to be applied.

1. What is the area of a rectangle having a width of 45 cm and a length of 12 cm?
2. What is the surface area of a sphere of radius 7.23 cm?
3. What is the area of the trapezoid?



4. What is the surface area of a cone having a slant height of 6.80 cm and a base radius of 5.20 cm?
5. What is the area of a circle of radius 6.00 cm?
6. What is the surface area of the pyramid? The slant height of the pyramid is 24.0 cm.



7. What is the surface area of a tetrahedron in which each face has an area of 12.5 cm^2 ?

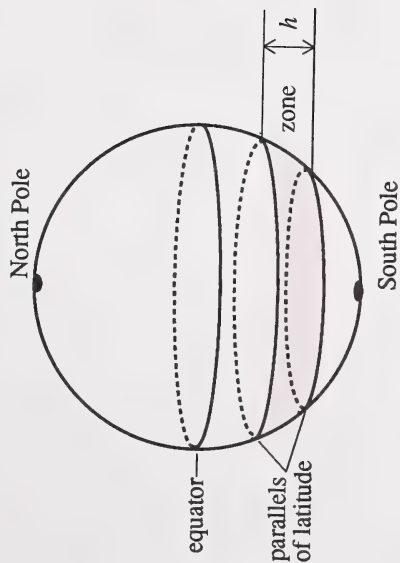


For solutions to **Extra Help**, turn to **Appendix A, Topic 2**.

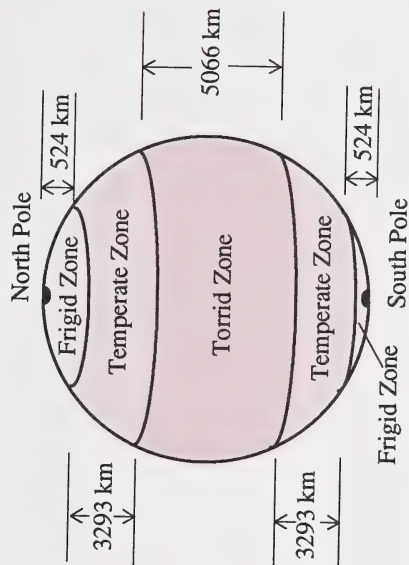


Extensions

Parallels of latitude are parallel to the equator. The region between two parallels of latitude is a zone. The area of a zone is given by $A = \pi dh$, where d is earth's diameter equal to 12 700 km, and h is the vertical distance between the parallels of latitude bordering the zone.



Using the following diagram, try the questions which follow.



1. What is the area of the Torrid Zone?
2. What is the area of each Temperate Zone?
3. What is the area of each of the Frigid Zones?
4. What percentage of earth's surface area is in the Temperate Zones?



For solutions to Extensions,
turn to **Appendix A, Topic 2.**

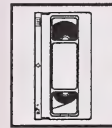
Topic 3 Volume



Introduction

The total volume through which the pistons of an engine move is called the engine displacement. For a chain saw, the engine displacement can be 50 cm^3 . For a motorcycle, the engine displacement can be 1500 cm^3 . For a large car, a common engine displacement is close to 6 L (6000 cm^3).

The volume through which the pistons move gives an indication of how much power the engine can deliver. In this topic, you will gain an understanding of volume and its application.



You may view the video titled *Mathways*¹, program 4 on volumes as an introduction to this topic.

¹ *Mathways* is a title of Wisconsin ETV Network.

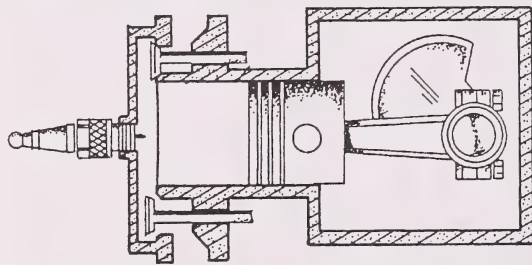


What Lies Ahead

Throughout this topic you will learn to

1. calculate the volume of prisms and pyramids
2. calculate the volume of cylinders and cones
3. solve problems involving the volume of spheres

Now that you know what to expect, turn the page to begin your study of volume.





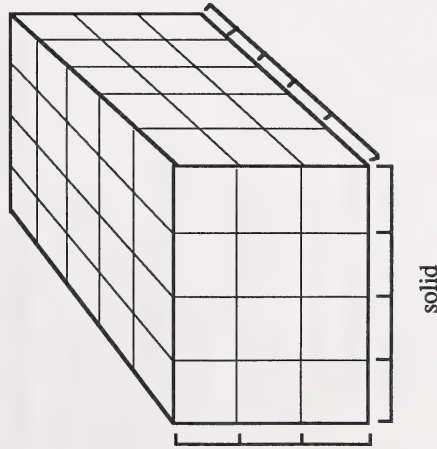
Exploring Topic 3

Activity 1



Calculate the volume of prisms and pyramids.

Volume is a measure of the number of cubic units in a solid.



There are 20 cubic units in the top layer of the solid. You can determine this much like you determine the area of a rectangle. There are three layers of cubic units, so the total number of cubic units contained in the solid is 3×20 cubic units or 60 cubic units. The volume of the solid is 60 cubic units.

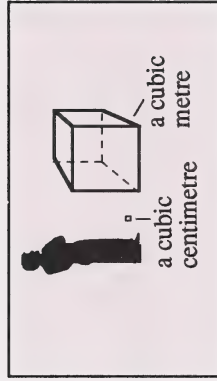


cubic unit

The cubic unit shown is a cube which has a length, width, and height equal to 1 cm. Such a cubic unit is one cubic centimetre or 1 cm^3 . Since there are sixty cubic centimetres contained in the solid, the volume of the solid is 60 cm^3 .

The solid above is a rectangular prism. For such a solid, $V = lwh$.

The cubic units in which the volume is expressed depends on the unit of measurement used for the dimensions. For example, when the length, width, and height are measured in centimetres, the volume is in cubic centimetres. When the dimensions are in metres, the volume is in cubic metres (1 m^3). A cubic metre is a cube measuring 1 m by 1 m by 1 m.



You may have started to wonder how many cubic centimetres there are in a cubic metre. You can find out by using the volume formula. Express the dimensions of the cubic metre in centimetres.

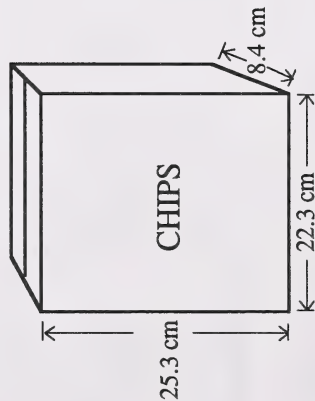
$$\begin{aligned} V &= lwh \\ &= (100 \text{ cm})(100 \text{ cm})(100 \text{ cm}) \\ &= 1\,000\,000 \text{ cm}^3 \text{ or } 1 \times 10^6 \text{ cm}^3 \end{aligned}$$

There are $1\,000\,000 \text{ cm}^3$ in a cubic metre.

Look at the following examples in which volumes are calculated.

Example 1

What is the volume of the box of potato chips? Round the answer to the nearest cubic centimetre.



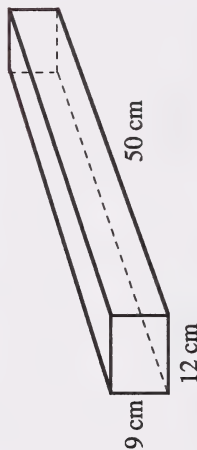
Solution:

$$\begin{aligned} V &= lwh \\ &= (22.3 \text{ cm})(8.4 \text{ cm})(25.3 \text{ cm}) \\ &\approx 4739 \text{ cm}^3 \end{aligned}$$

The volume of the box is 4739 cm^3 .

Example 2

Calculate the volume of the solid.

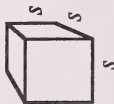


Solution:

$$\begin{aligned} V &= lwh \\ &= (50 \text{ cm})(12 \text{ cm})(9 \text{ cm}) \\ &= 5400 \text{ cm}^3 \end{aligned}$$

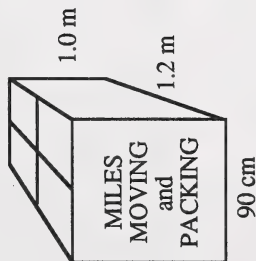
The volume of the solid is 5400 cm^3 .

For a cube, the volume formula simplifies to $V = s^3$.



Example 3

Calculate the volume of the packing box.



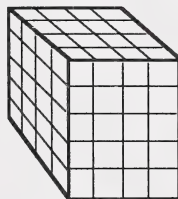
Solution:

$$\begin{aligned} V &= lwh \\ &= (1.2 \text{ m})(0.90 \text{ m})(1.0 \text{ m}) \\ &= 1.08 \text{ m}^3 \end{aligned}$$

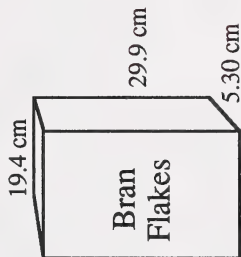
The volume of the box is 1.08 m^3 .

Try the following questions.

1. The small cubes making up the solid are cubic centimetres. What is the volume of the solid?



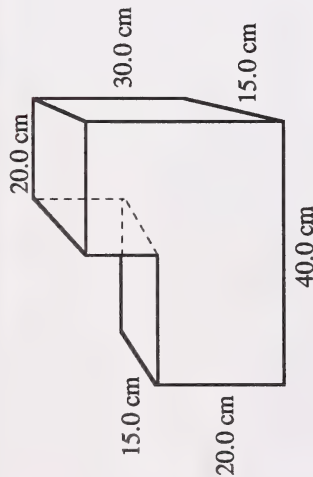
2. What is the volume of the cereal box? Round your answer to the nearest cubic centimetre.



3. According to the manufacturer, a portable humidifier is adequate for a space of 50 m^3 . Suppose the humidifier were used in a room with a height of 3.0 m and a floor measuring 3.6 m by 4.5 m. Using the manufacturer's criteria, determine whether the humidifier would be able to maintain the proper room humidity.

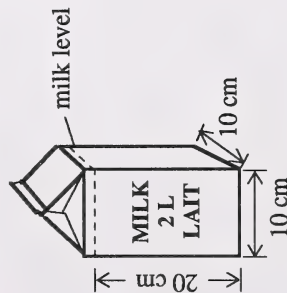
All dimensions must be expressed in the same units.

4. Calculate the volume of the solid.



For solutions to Activity 1, turn to **Appendix A, Topic 3.**

In a two-litre milk carton, the milk level is 20 cm above the base of the carton. The base measures 10 cm by 10 cm. The volume of milk is, therefore, $10 \text{ cm} \times 10 \text{ cm} \times 20 \text{ cm} = 2000 \text{ cm}^3$.



This shows that 2 L equals 2000 cm^3 . There are 1000 mL (1000 millilitres) in a litre. It follows that

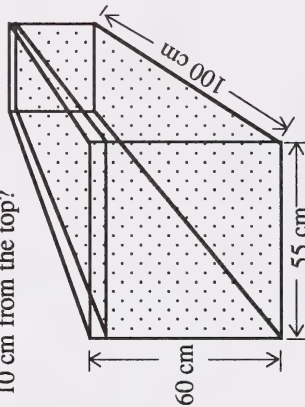
$$2000 \text{ mL} = 2000 \text{ cm}^3$$

$$1 \text{ mL} = 1 \text{ cm}^3$$

Generally, fluids are measured in litres. Litres may also be used when the volume in cm^3 becomes large numerically. For example, an engine displacement of 6500 cm^3 would be expressed as 6.5 L.

Try the following questions. Do at least one.

5. How many litres of water are required to fill the aquarium so that the water level is 10 cm from the top?



6. How many litres are there in 1 m^3 ?



For solutions to Activity 1, turn to **Appendix A, Topic 3.**

Hint: Think of the solid as the sum of two rectangular prisms.

The symbol for litre is L.

You may have found that determining the volume was like multiplying the area of one surface by the height of the solid. This idea is reflected in the volume formula.

$$\begin{aligned} V &= lwh \\ &= (lw)h \end{aligned}$$

If you consider lw to be the area of the base (B), the formula becomes

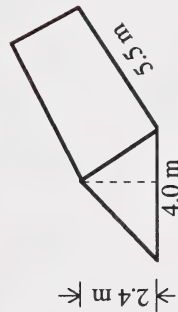
$$V = Bh.$$

This is a more general formula that can be applied to any prism.

Look at the following examples.

Example 4

What is the volume of the solid?



Solution:

$$V = Bh$$

$$B = \frac{1}{2}bh$$

$$= \frac{1}{2}(4.0 \text{ m})(2.4 \text{ m})$$

$$= 4.8 \text{ m}^2$$

$$V = (4.8 \text{ m}^2)(5.5 \text{ m})$$

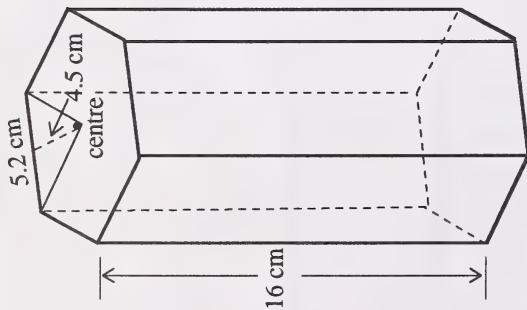
$$= 26.4 \text{ m}^3$$

The volume of the solid is 26.4 m^3 .

Either end of a prism can be considered to be the base.

Example 5

A utility cloth manufacturer uses packaging in the shape of a regular hexagonal prism as shown. Find the volume of the prism.



Solution:

$$\text{Base} = \frac{1}{2} \text{ans}$$

$$= \frac{1}{2}(4.5 \text{ cm})(6)(5.2 \text{ cm})$$

$$= 70.2 \text{ cm}^2$$

$$V = Bh$$

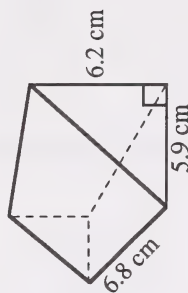
$$= (70.2 \text{ cm}^2)(16 \text{ cm})$$

$$= 1123 \text{ cm}^3$$

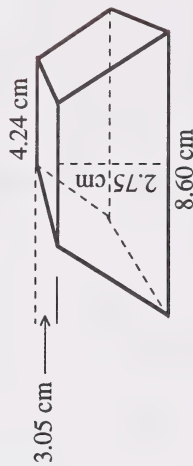
The volume of the prism is 1123 cm^3 .

Try the following questions. Do at least the even-numbered questions.

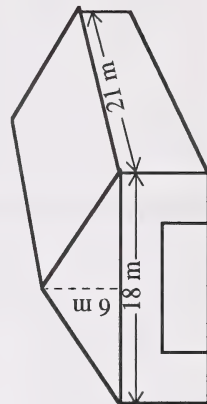
7. What is the volume of the triangular prism? Give your answer to the nearest cubic centimetre.



8. What is the volume of the prism?

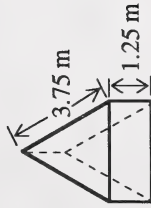


9. What is the volume of the air above the ceiling, but below the roof of the shed?



10. What is the volume of the prism? The ends are equilateral triangles.

Hint: See Example 10 of Topic 2 for the area of an equilateral triangle.



For solutions to Activity 1, turn to Appendix A, Topic 3.



There is a simple volume relationship between a pyramid and a square prism with the same base and height. This can be discovered by comparing the amount each figure can hold.

For nets for Activity 1, turn to Appendix B, nets 11 and 12.

Try the following question using the nets.

11. Trace the net for the square prism onto cereal box cardboard. Then, use the net to make a container in the shape of a square prism. Use tape to secure the joints.

Trace the triangle four times onto cereal box cardboard. Join the cut-out triangles with tape in order to make a pyramid-shaped container.

The pyramid has the same height and base of the square prism.

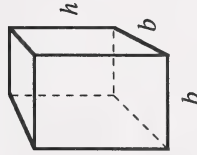
Use rice to compare the volumes of the figures. Determine how many pyramids full of rice can be put into the prism. Tap the pyramid to settle the rice as you fill it. Also, tap the prism as you transfer the rice.



For solutions to Activity 1, turn to Appendix A, Topic 3.

You have discovered that you can empty three pyramids of rice into the prism.

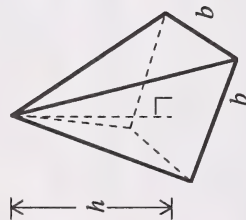
For a square prism, $V = b^2 h$.



The pyramid has one-third of this volume. For a pyramid, $V = \frac{1}{3}b^2h$ or

$$V = \frac{1}{3}Bh, \text{ where}$$

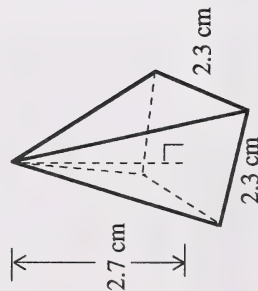
B is the area of the base.



Look at the examples.

Example 6

What is the volume of the figure?



Solution:

$$A = \frac{1}{3}b^2h$$

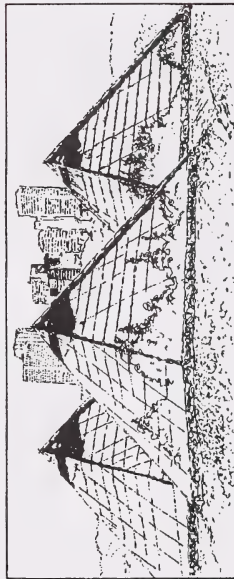
$$= \frac{1}{3}(2.3 \text{ cm})^2(2.7 \text{ cm})$$

$$\approx 4.8 \text{ cm}^3$$

The volume is 4.8 cm^3 .

Example 7

The show pyramid of the Muttart Conservatory is 18.0 m high. The pyramid's floor area is 381 m^2 . Determine the volume of air inside the structure.



Solution:

$$V = \frac{1}{3}Bh$$

$$= \frac{1}{3}(381 \text{ m}^2)(18.0 \text{ m})$$

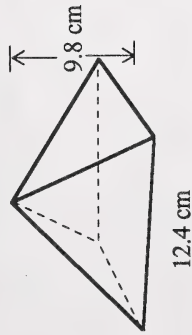
$$= 2286 \text{ m}^3$$

The volume of air is 2286 m^3 .

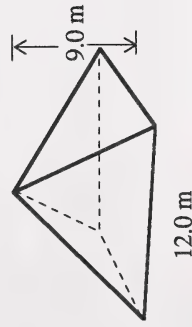
In the volume formula for a pyramid, the h stands for the perpendicular distance from the base to the top of the pyramid. The distance h is not the slant height.

Try the following questions. Do at least the even-numbered questions.

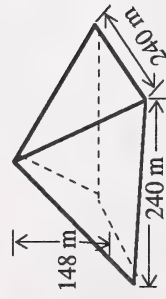
12. What is the volume of the pyramid?



13. What is the volume of the pyramid?

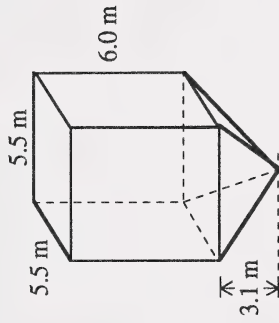


14. What is the volume of the pyramid?

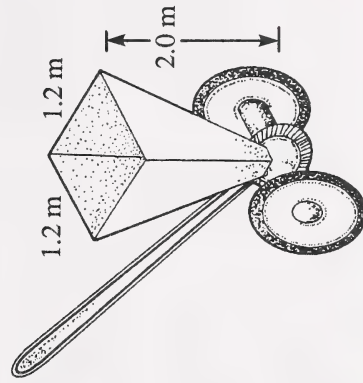


15. The temperate pyramid of the Muttart Conservatory is 24.1 m high. Its floor forms a square measuring 25.7 m by 25.7 m. What is the volume of this pyramid?

16. A grain hopper has the dimensions as shown. What is the volume of the container?



17. What is the volume of the hopper in the fertilizer spreader? Assume the hopper has the shape of a pyramid.



18. What is the length of each side of a cube having a volume of 25.0 m^3 ?



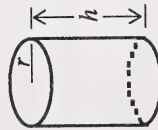
For solutions to Activity 1, turn to Appendix A, Topic 3.

Activity 2



Calculate the volume of cylinders and cones.

For a prism, you found that the volume is a product of the area of one end and the height. You can think of the area of one end as giving the number of cubic units in one layer. The height indicates the number of layers. This is also true for cylinders. The formula $V = Bh$ applies.



For a cylinder, the ends are circular and $A = \pi r^2$. For the cylinder, the formula becomes

$$V = Bh$$

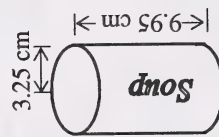
$$V = (\pi r^2)h$$

$$V = \pi r^2 h.$$

Look at the examples.

Example 8

What is the volume of a soup can having a radius of 3.25 cm and a height of 9.95 cm ?



Solution:

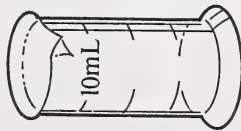
$$V = \pi r^2 h$$

$$\begin{aligned} &= (\pi)(3.25 \text{ cm})^2(9.95 \text{ cm}) \\ &\approx 330 \text{ cm}^3 \end{aligned}$$

The volume of the can is 330 cm^3 .

Example 9

Suppose a manufacturer of measuring cylinders had to decide where to place the 10 mL marking. If the cylinders have an inside diameter of 1.10 cm, how high from the bottom should the marking be placed?



Solution:

$$r = \frac{1.10}{2} \text{ cm}$$

$$= 0.550 \text{ cm}$$

$$V = 10 \text{ mL}$$

$$= 10 \text{ cm}^3$$

$$V = \pi r^2 h$$

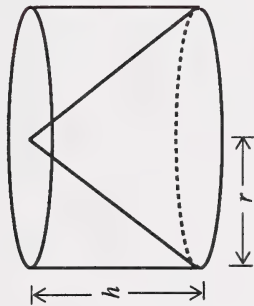
$$h = \frac{V}{\pi r^2}$$

$$= \frac{10 \text{ cm}^3}{(\pi)(0.550 \text{ cm})^2}$$

$$\approx 10.52 \text{ cm}$$

The marking should be placed at 10.52 cm.

In the experiment with rice, you found that a factor of $\frac{1}{3}$ applies to the volume relationship between a rectangular solid and the associated pyramid. This factor also applies when comparing volumes of a cone and a cylinder. The volume of a cone is one-third that of a cylinder having the same base and height.



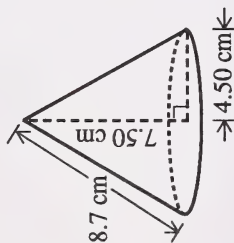
The volume of a cylinder can be found by using the formula $V = \pi r^2 h$.

The volume of a cone can be found by using the formula $V = \frac{1}{3} \pi r^2 h$.

Remember that h refers to the perpendicular height from the base to the point of the cone. Slant height, s , is used for surface area.

Example 10

Determine the volume of the cone.



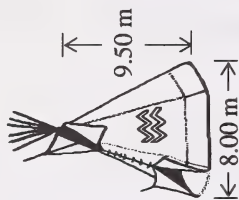
Solution:

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} (\pi) (4.50 \text{ cm})^2 (7.50 \text{ cm}) \\ &\approx 159 \text{ cm}^3 \end{aligned}$$

The volume of the cone is 159 cm^3 .

Example 11

What is the volume of the space enclosed by the teepee? Assume the base is circular.



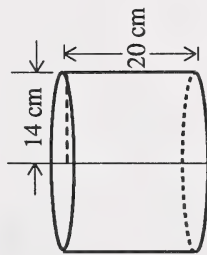
Solution:

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} (\pi) (4.00 \text{ m})^2 (9.50 \text{ m}) \\ &\approx 159 \text{ m}^3 \end{aligned}$$

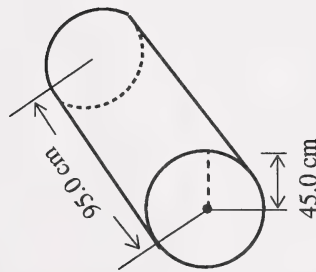
The volume of the teepee is 159 m^3 .

Try the following questions. Do at least the odd-numbered questions.

1. What is the volume of the cylinder?



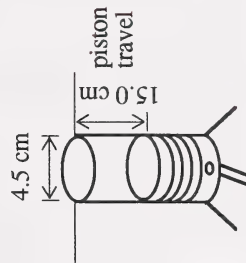
2. Determine the cylinder volume.



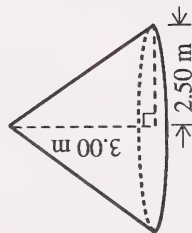
3. To what height would the liquid rise if 10 L of water were poured into the container?



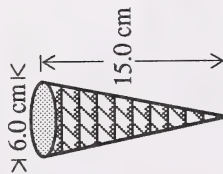
4. What is the displacement of a one-cylinder engine in which the piston is as shown?



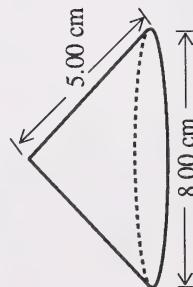
5. What is the volume of the cone?



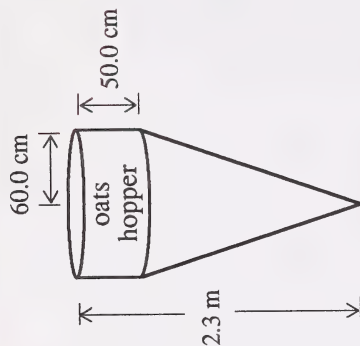
6. The ice cream treat is cone-shaped. What volume is this cone?



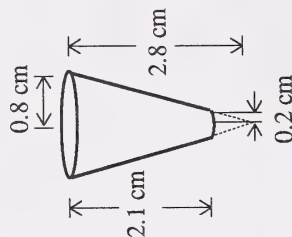
7. Determine the volume of the cone.



8. What is the volume of the hopper?

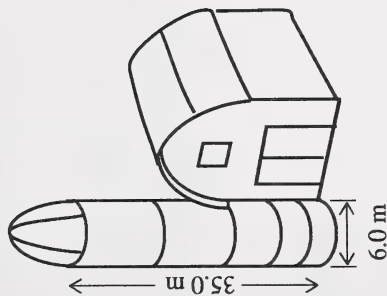


9. The figure below forms part of a cone having a height of 2.8 cm. The figure itself has a height of 2.1 cm. The radius of one end is 0.8 cm, and the radius of the other is 0.2 cm. What is the volume of the figure?



Use the Theorem of Pythagoras to determine the height of the cone.

10. Assume that silage can be stored to a height of 35.0 m in a silo measuring 6.0 m across. What volume of silage can be stored?



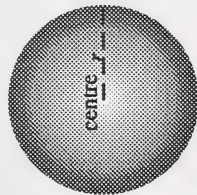
For solutions to Activity 2, turn to Appendix A, Topic 3.

Activity 3



Solve problems involving the volume of spheres.

The volume of a sphere is given by the formula $V = \frac{4}{3} \pi r^3$.



Example 12

Determine the volume of the sphere.

Solution:

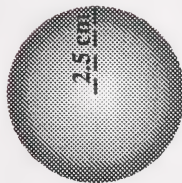
$$r = 2.5 \text{ cm}$$

$$V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} (\pi) (2.5 \text{ cm})^3$$

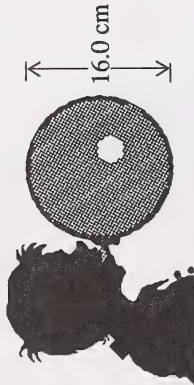
$$\approx 65 \text{ cm}^3$$

The volume of the sphere is 65 cm^3 .



Example 13

How much space is there inside the balloon?



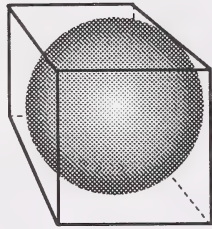
Solution:

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} (\pi) (8.0)^3 \\ &\approx 2145 \text{ cm}^3 \end{aligned}$$

There is 2145 cm^3 of space inside the balloon.

Example 14

How much space is there inside the cube and outside the sphere? The cube has side lengths of 1.00 m and the sphere is the largest possible inside the cube.



Solution:

For the cube,

$$\begin{aligned} V &= lwh \\ &= (1.00 \text{ m})(1.00 \text{ m})(1.00 \text{ m}) \\ &= 1.00 \text{ m}^3 \end{aligned}$$

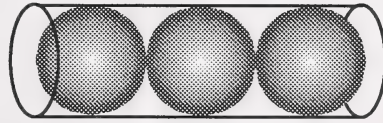
For the sphere, the radius is 0.50 m, and

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} (\pi) (0.50 \text{ m})^3 \\ &= 0.52 \text{ m}^3 \end{aligned}$$

The volume inside the cube and outside the sphere is $1.00 \text{ m}^3 - 0.52 \text{ m}^3 = 0.48 \text{ m}^3$.

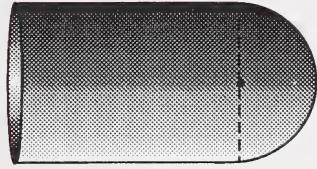
Try the following questions.

1. What is the volume of a sphere of radius 0.75 m?
2. What is the volume of a beach ball having a diameter of 1.15 m?
3. The cylindrical container has three tennis balls in it. The balls and the cylinder have a diameter of 6.50 cm. The cylinder is just tall enough so that the balls fit snugly. How much air is there in the container and outside the balls?



4. What is the radius of a sphere having a volume of 113 cm^3 ? Give your answer to the nearest centimetre.
5. How does doubling the radius affect the volume of a sphere? You may use radii of 1 m and 2 m to support your answer.

6. The container is composed of a cylinder and a hemisphere of radius 4.0 m. The height of the cylinder above the hemisphere is 11 m. What is the volume of the container?

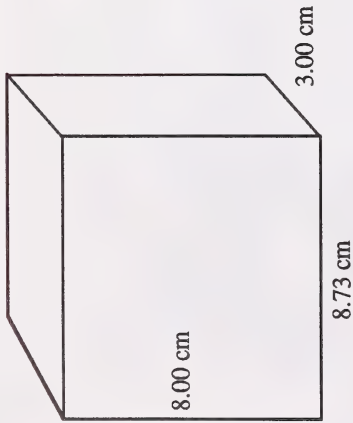


For solutions to Activity 3, turn to **Appendix A, Topic 3.**

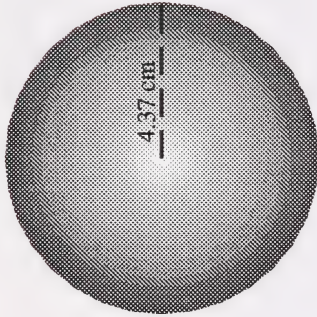
You have studied the surface area and the volume of a variety of solids. One question may come to mind: What shape must solids, with a fixed surface area, have for the greatest volume? You can explore this in the next questions.

7. Use the chart which follows to record your work for parts a and b.
 - a. Determine the surface area of each of the following solids.
 - b. Determine the volume of each of the following solids.
 - c. Which solid has the greatest volume?

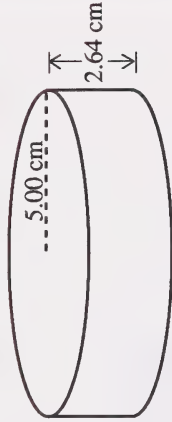
Do you remember the problem of enclosing the greatest area for a given perimeter? The problem here is similar except that it is in three dimensions rather than two.



Rectangular Prism



Sphere

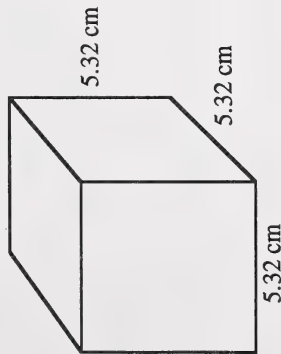


Cylinder

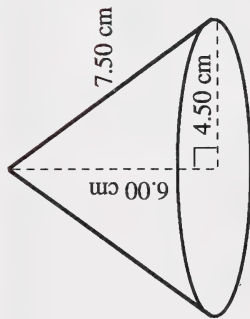
	Rectangular Prism	Sphere	Cylinder
a. Surface area			
b. Volume			

8. Use the chart which follows to record your work for parts a and b.

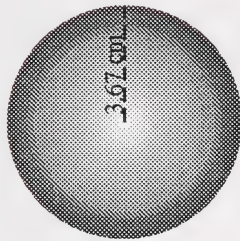
- Determine the surface area of each of the following solids.
- Determine the volume of each of the following solids.
- Which solid has the greatest volume?



Cube



Cone



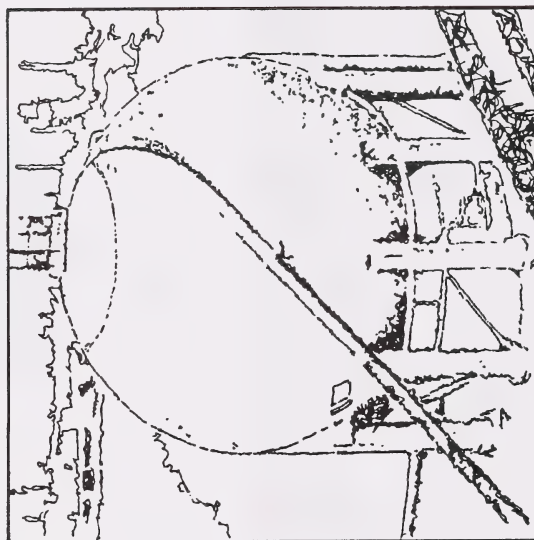
Sphere

	Cube	Cone	Sphere
a. Surface area			
b. Volume			

9. Based on your limited experience, which solid would you say has the greatest volume for a given surface area?



For solutions to Activity 3, turn to Appendix A, Topic 3.



You have seen that the sphere has more volume for surface area than any other shape. This fact is useful in designing tanks. Less material is needed to build a spherical tank for a given capacity than any other shape.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.



Extra Help

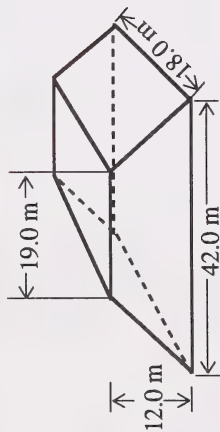
The formulas in the table can be used to find the volume of a solid.

Figure	Formula for Volume
Rectangular Prism	$V = lwh$
Prism	$V = Bh$
Pyramid	$V = \frac{1}{3}b^2h$

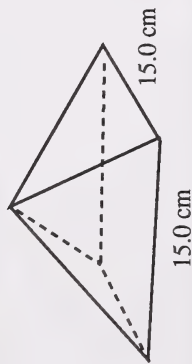
Figure	Formula for Volume
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{1}{3}\pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$

Try the following questions.

1. What is the volume of the prism?



2. What is the volume of the prism? The height of the prism is 9.0 cm.



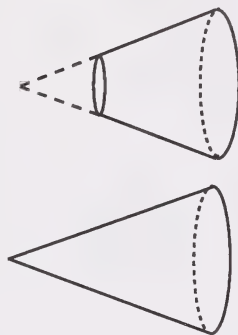
3. What is the volume of a cylinder having a height of 1.00 m and a radius of 0.500 m?
4. What is the volume of a cone which has a height of 12.0 cm and a circular base of area 75.0 cm^2 ?
5. What is the volume of a sphere of radius 6.00 m?

For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.



Extensions

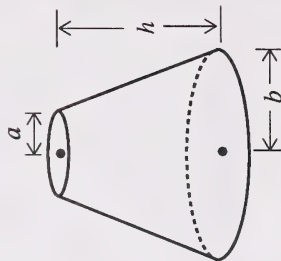
The top of a cone can be removed by a cut parallel to the base. What remains is called a **frustum**.



The volume of a frustum can be found using the formula

$$V = \frac{1}{3} \pi h (a^2 + ab + b^2),$$

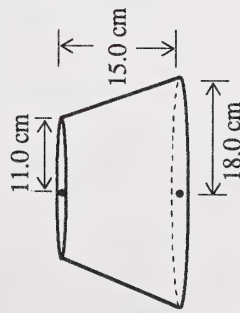
where h is the perpendicular distance from the base to the other flat surface,
 a is the radius of one of the flat surfaces, and
 b is the radius of the other flat surface.



Look at the example.

Example 15

What is the volume of the frustum?



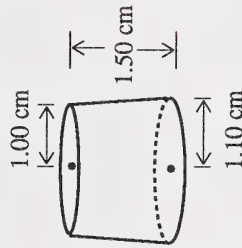
Solution:

$$\begin{aligned}
 V &= \frac{1}{3} \pi h (a^2 + ab + b^2) \\
 &= \frac{1}{3} (\pi) (15.0 \text{ cm}) \left[(11.0 \text{ cm})^2 + (11.0 \text{ cm})(18.0 \text{ cm}) + (18.0 \text{ cm})^2 \right] \\
 &\doteq 15.7 \text{ cm} \left[121 \text{ cm}^2 + 198 \text{ cm}^2 + 324 \text{ cm}^2 \right] \\
 &\doteq 15.7 \text{ cm} \left[643 \text{ cm}^2 \right] \\
 &\doteq 10\,095 \text{ cm}^3
 \end{aligned}$$

The volume of the frustum is $10\,095 \text{ cm}^3$.

Try the following questions.

1. What is the volume of this frustum?



2. Let $a = 0$ in $V = \frac{1}{3} \pi h (a^2 + ab + b^2)$. To which solid does the new formula apply?
3. Let $a = b$ in $V = \frac{1}{3} \pi h (a^2 + ab + b^2)$. To which solid does the new formula apply?



For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

Topic 4 Relationships for Geometric Figures



Introduction

These leaves have different sizes, shapes, and edges. What these leaves have in common is the pattern of the veins. All the leaves display a branch-like venation.



This topic is not about differences in geometric figures; it is about what geometric figures have in common. Just as these leaves have a venation pattern in common, so are there relationships that apply to many geometric figures. This topic deals with these common relationships.



What Lies Ahead

Throughout this topic you will learn to

1. determine and use the relationship between the sum of the angles and the number of sides in a polygon
2. determine and use the relationship between the number of sides and the number of diagonals in a polygon
3. determine the relationship among the numbers of edges, vertices, and faces in a polyhedron
4. determine the relationship between the number of lateral faces and the number of diagonals in a prism

Now that you know what to expect, turn the page to begin your study of relationships for geometric figures.



Exploring Topic 4

Activity 1

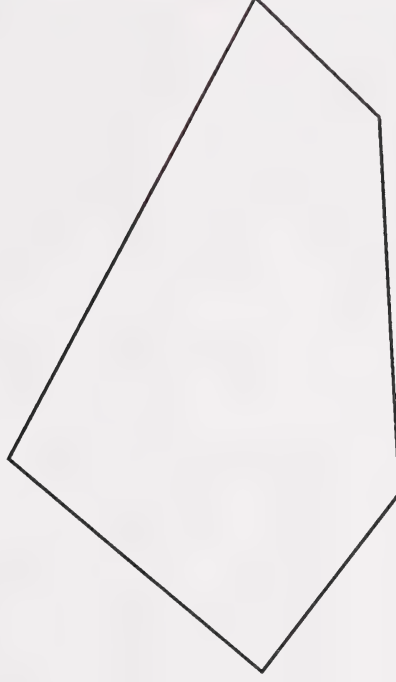
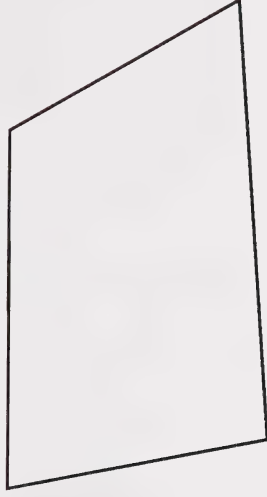


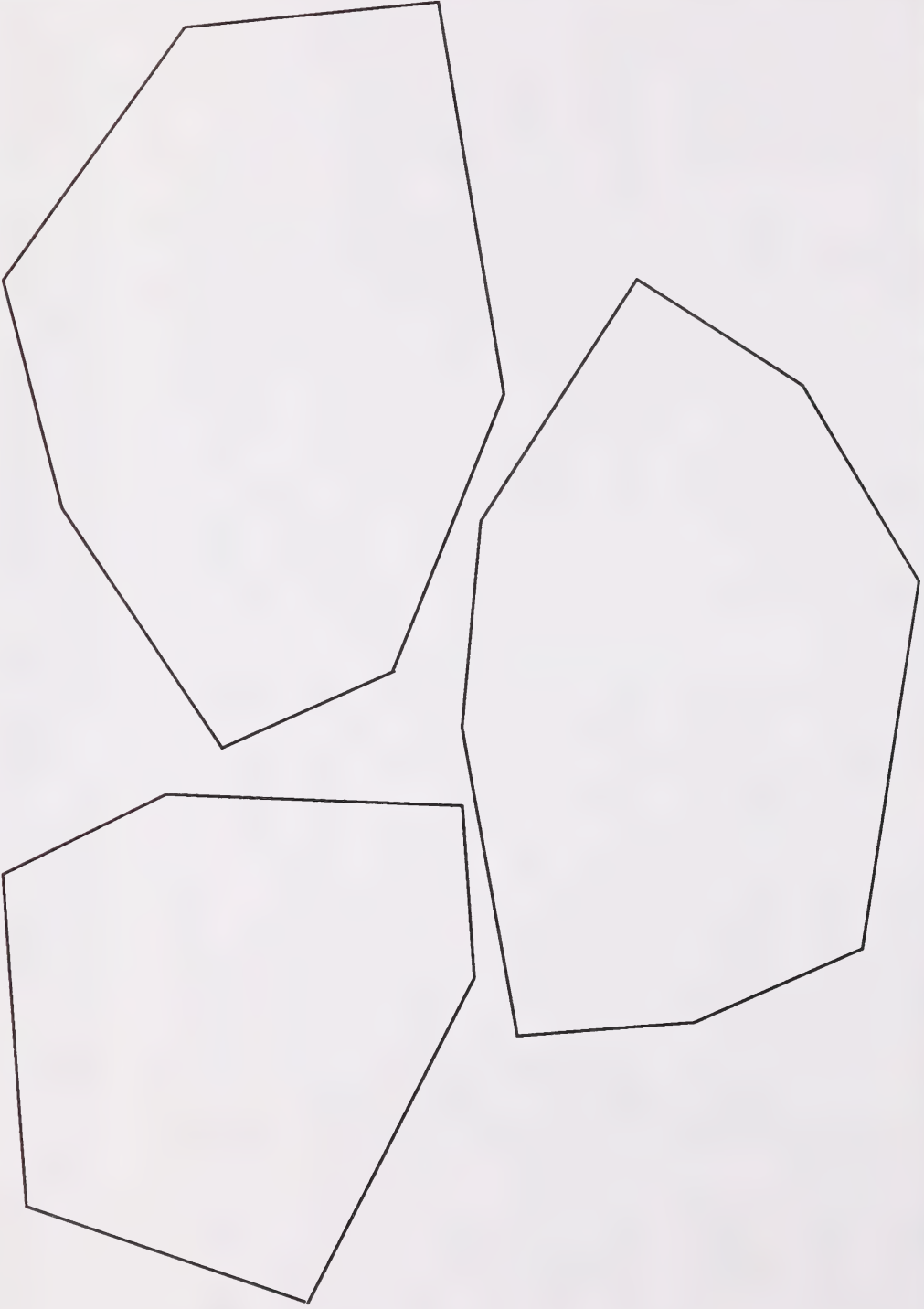
Determine and use the relationship between the sum of the angles and the number of sides in a polygon.

You already know that for the triangle, the angle sum is 180° . How does the angle sum increase as you increase the number of sides in a polygon? The next questions have you study the pattern. From this activity, you may determine how the angle sum is related to the number of sides.

Try the following questions.

These figures are needed for the questions.





- For each figure, measure the angles, add your measurements, and then determine the angle sum. Then complete the first three columns of the table.

Figure	Number of Sides	Angle Sum ° (degrees)	Alternate Expression of Angle Sum
Triangle	3	180°	
Quadrilateral			
	5		
	6		
Heptagon			

As soon as you can state the relationship between the number of sides and the angles sum, go directly to question 4.

- How does the angle sum increase as you increase the number of sides?
- For the triangle, the angle sum can be expressed as $180^\circ (3 - 2)$. For the quadrilateral the angle sum is $180^\circ (4 - 2)$. Fill in the blank in $180^\circ (\underline{\hspace{1cm}} - 2)$ for the other angle sums. You can call these alternate expressions of angle sums. Use these to fill in the last column of the table.
- Based on your observations, complete the following statement.
The sum of the angles of a polygon of n sides is $\underline{\hspace{2cm}}$.



For solutions to Activity 1,
turn to **Appendix A, Topic 4**.

Use a mathematical expression involving the n to complete the statement.

You have discovered that for a polygon, the angle sum is equal to $180^\circ(n - 2)$, where n is the number of sides. The examples show how this relation can be applied.

Example 1

Find the sum of the angles for an 18-sided polygon.

Solution:

$$\begin{aligned}\text{Angle sum} &= 180^\circ(n - 2) \\ &= 180^\circ(18 - 2) \\ &= 180^\circ(16) \\ &= 2880^\circ\end{aligned}$$

The angle sum is 2880° .

Example 2

How many sides does a polygon have if the angle sum is 5220° ?

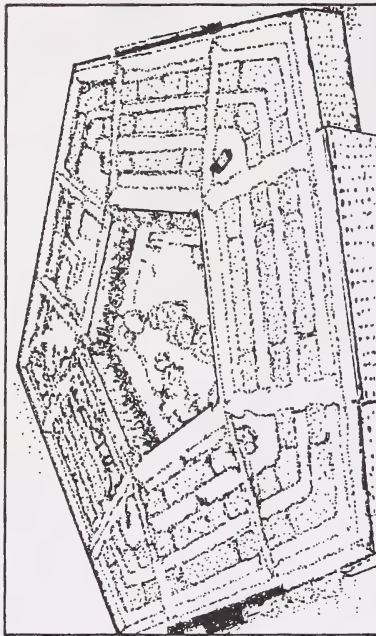
Solution:

$$\begin{aligned}\text{Angle sum} &= 180^\circ(n - 2) \\ 5220^\circ &= 180^\circ(n - 2) \\ n - 2 &= \frac{5220^\circ}{180^\circ} \\ n - 2 &= 29 \\ n &= 29 + 2 \\ n &= 31\end{aligned}$$

The polygon has 31 sides.

Example 3

The building is in the shape of a regular pentagon. At what angle do the outer walls meet?



Solution:

For a polygon, the
angle sum $= 180^\circ(n - 2)$.

$$\begin{aligned}\text{For a pentagon, the} \\ \text{angle sum} &= 180^\circ(5 - 2) \\ &= 180^\circ(3) \\ &= 540^\circ.\end{aligned}$$

The five angles are equal and each angle measures
 $(\frac{1}{5})(540^\circ) = 108^\circ$.

The outer walls meet at an angle of 108° .

Try the following questions. Do at least the odd-numbered questions.

5. In each of a, b, c, d, and e, the number of sides of a polygon is given. Determine the sum of the angles in each case.

- a. 10 b. 4 c. 50
d. 16 e. 20

6. In each of a, b, c, d, and e, the angle sum of a polygon is given. Determine the number of sides of each figure.

- a. 1800° b. 2160° c. 360°
d. 180° e. 1620°

7. Five angles of a hexagon are 125° , 145° , 95° , 112° , and 81° . Calculate the measure of the remaining angle.

8. Three angles of a quadrilateral are 85° , 105° , and 97° . What is the measure of the fourth angle?

9. What is the measure of each of the angles of a regular decagon?

10. What is the measure of each of the angles of a regular heptagon? Round your answer to the nearest degree.



For solutions to Activity 1, turn to Appendix A, Topic 4.

Activity 2

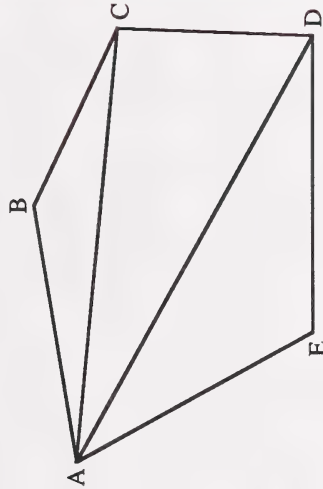


Determine and use the relationship between the number of sides and the number of diagonals in a polygon.

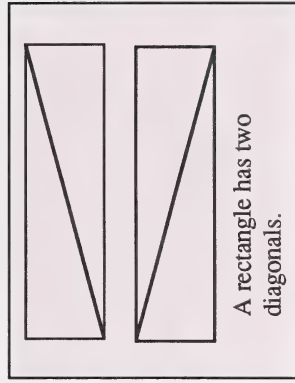
A diagonal of a rectangle is a line segment joining opposite vertices. The term diagonal can be applied to other polygons with the following definition:

A diagonal of a polygon is a line segment other than a side of the polygon that joins two vertices of the polygon.

Can you identify diagonals in the figure?



In a regular polygon, the angles are all the same size.

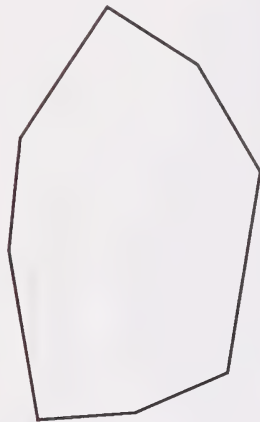
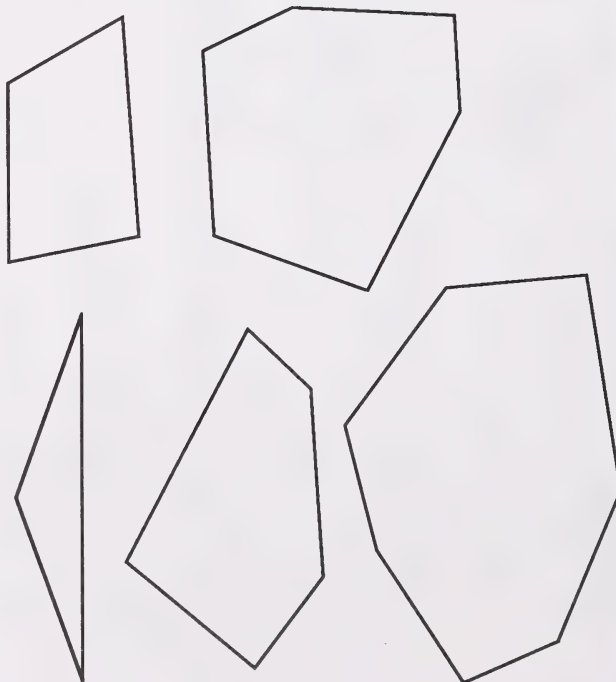


In the pentagon, $\overline{AD}$ is a diagonal. $\overline{AB}$ joins vertex A and vertex B, but $\overline{AB}$ is not a diagonal since it is a side. Another diagonal is $\overline{AC}$. You can draw in $\overline{BE}$, $\overline{BD}$, and $\overline{CE}$ to display the remaining diagonals of the pentagon.

The next questions have you determine for a polygon the relationship between the number of sides and the number of diagonals.

Try the following questions.

Use the figures for the questions.



As soon as you determine a formula for the number of diagonals, go directly to question 4.

- For each of the figures, draw the diagonal and complete the appropriate entries in the first two columns.

Figure	Number of Sides	Number of Diagonals	Alternate Expression of Number of Diagonals
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
Octagon			

2. For the triangle, the number of diagonals equals $\frac{(3)(0)}{2}$. For the quadrilateral, the number of diagonals is $\frac{(4)(1)}{2}$. For the pentagon, the number of diagonals is $\frac{(5)(2)}{2}$. See how the factors in the numerator increase by 1 each time. Fill in the blanks in $\frac{(\quad)(\quad)}{2}$ for the other numbers of diagonals.

Use these expressions to fill in the last column of the table.

3. For each polygon, how do the factors in the numerator of the alternate expression compare to the number of sides?

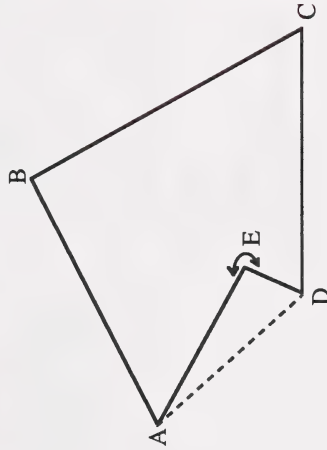
4. Based on your observations, complete the following statement.

The number of diagonals in a polygon of n sides is _____.

You have discovered that the number of diagonals in an n -sided polygon is $\frac{n(n-3)}{2}$.

There is a restriction on the use of this formula as it only applies to convex polygons.

A **convex** polygon is a polygon in which all the angles are less than 180° . In a convex polygon, no line segment joining any of the vertices goes through the exterior of the polygon.

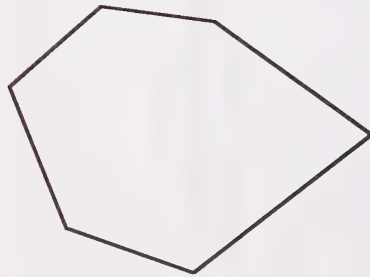


This figure is **not** convex. The angle of the polygon at E is greater than 180° .



For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

Hint: Complete the statement with a mathematical expression involving the variable n .



This figure is convex. Each angle is less than 180° .

For figure ABCDE, a line segment joining A and D is in the exterior of the figure. It is not natural to consider this segment a diagonal of the figure. Yet, this segment is "counted" by $\frac{n(n-3)}{2}$. Using the formula, you would get 1 too many. To avoid this problem, the formula is said to apply only to convex polygons. When you are to determine the number of diagonals in a polygon, you may assume the polygon is convex, even when it is not explicitly stated.

Now you can see how the formula is applied. See the next examples.

Example 4

How many diagonals does a polygon of 20 sides have?

Solution:

$$\begin{aligned}\text{Number of diagonals} &= \frac{n(n-3)}{2} \\ &= \frac{20(20-3)}{2} \\ &= 170\end{aligned}$$

Example 5

A convex polygon has 20 diagonals. How many sides does the polygon have?

Solution:

$$\text{Number of diagonals} = \frac{n(n-3)}{2}$$

$$20 = \frac{n^2 - 3n}{2}$$

$$2(20) = n^2 - 3n$$

$$40 = n^2 - 3n$$

$$n^2 - 3n - 40 = 0$$

$$(n-8)(n+5) = 0$$

$$n-8=0 \text{ or } n+5=0$$

$$n=8 \text{ or } n=-5$$

$$n=-5 \text{ is not a possible value for the problem.}$$

The polygon has eight sides.

Try the following questions. Do at least the odd-numbered questions.

5. In each of a, b, c, and d, the number of sides of a polygon is given. Determine the number of diagonals in each case.

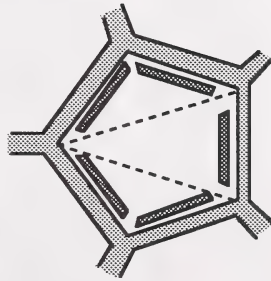
a. 9 b. 10
c. 14 d. 20

6. In each of a, b, c, and d, the number of sides of a polygon is given. Determine the number of diagonals in each case.

a. 12 b. 13
c. 15 d. 16

7. A polygon has 44 diagonals. How many sides does it have?

8. Five sidewalks converge onto a pentagonal lawn surrounded by sidewalk. How many straight paths can be taken across the central lawn to connect the converging sidewalks?



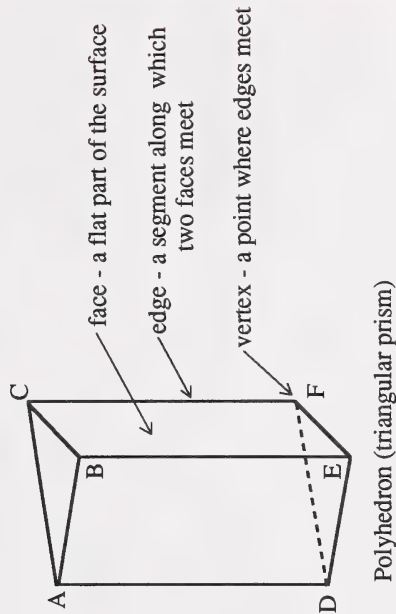
For solutions to Activity 2, turn to Appendix A, Topic 4.

Activity 3



Determine the relationship among the numbers of edges, vertices, and faces in a polyhedron.

Now you can return to the polyhedron. There is a relationship for the edges, vertices, and faces of a polyhedron. These terms are illustrated as follows:



The five faces are the polygons ABC, DEF, ABED, BCFE, and ACFD. The nine edges are $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{DE}$, $\overline{EF}$, $\overline{DF}$, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$. The six vertices are labelled on the diagram.

There is a relationship among numbers of edges, vertices, and faces that applies to all polyhedrons. You may discover this for yourself in the next questions.

Use the table for the questions.

Figure	Number of Faces (F)	Number of Vertices (V)	Number of Edges (E)
Pyramid			
Rectangular prism			
Triangular prism			
Regular tetrahedron			
Regular hexahedron (cube)			
Regular octahedron			
Figure A			
Figure B			
Figure C			

- Fill in the table for the first six figures. To help you, use diagrams or the models you made earlier in this unit or use the diagrams in Unit 2, Topic 4.

As soon as you have determined the relation among F, V, and E, go to question 5.

- Complete the table for Figure A.

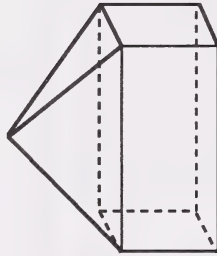


Figure A

- Complete the table for Figure B. The figure shows a cement block with one step in it.

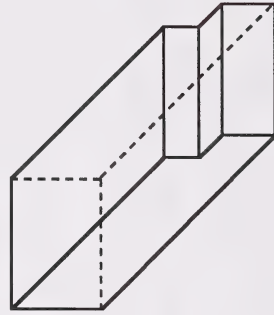


Figure B

4. A certain manufacturer markets disposable cloths in hexagonal prism-shaped containers as shown. Complete the table for Figure C.

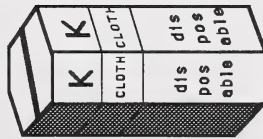


Figure C

5. Write an equation to show how F , V , and E are related.

You have discovered that for a polyhedron

$$F + V - E = 2.$$

Apply this relation in the next question.

6. A rhombicosidodecahedron is a polyhedron with 62 faces and 60 vertices. How many edges does it have?

For solutions to **Activity 3**, turn to **Appendix A, Topic 4**.

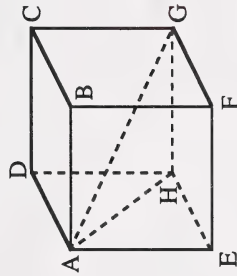


Activity 4



Determine the relationship between the number of lateral faces and the number of diagonals in a prism.

For a polyhedron, a diagonal is a line segment not in any face, but joining two vertices. For example, in the rectangular prism, $\overline{AD}$ and $\overline{AH}$ join vertices, but are not diagonals of the polyhedron since these segments lie in a face.



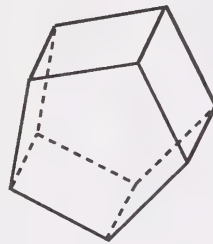
The segments $\overline{AG}$, $\overline{BH}$, $\overline{CE}$, and $\overline{DF}$ are diagonals. These join vertices, but do not have any face.

$F + V - E = 2$ is also known as Euler's Formula.

Note: When there is a reference to diagonals of a prism, assume that the prism is convex.

1. Complete the row for the rectangular prism.

2.



- Use the diagram to determine the number of diagonals in a pentagonal prism. Then fill in the related entries in the table.
- Notice that the diagonals always run from one end to the other.
3. Determine the number of diagonals in a hexagonal prism and complete the next row of the table.

Hint: Look at the number of diagonals from any one vertex. This may help you determine the total number of diagonals.

4. How does the number of diagonals in a prism compare to the number of diagonals in a polygon that represents the shape of the prism end? (Compare to your work in Activity 2.)

5. Complete the statement:

The number of diagonals in a prism having n lateral faces is _____.



For solutions to Activity 4, turn to **Appendix A, Topic 4.**

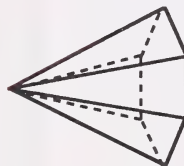


Figure C

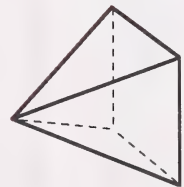


Figure B



Figure A

None of these polyhedrons has any diagonal. For each, a line segment joining any two of the vertices always lies in a side. Figure C is like a pyramid, except that it has a hexagonal base.

Convex prisms have convex ends. For convex prisms there is a relationship between the number of lateral faces and the number of diagonals. The lateral faces are those that are perpendicular to the ends. The triangular prism has three lateral faces and no diagonals. The rectangular prism has four lateral faces and four diagonals.

The next questions deal further with the relationship.

Prism Type	Number of Lateral Faces (n)	Number of Diagonals
Triangular prism		
Rectangular prism		
Pentagonal prism		
Hexagonal prism		
Heptagonal prism		

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

The angle sum for an n -sided polygon is $180(n - 2)$. The number of diagonals in an n -sided polygon is $\frac{1}{2}n(n - 3)$. The number of diagonals in a prism having n lateral faces is $n(n - 3)$.

The relationship among the numbers of faces (F), vertices (V), and edges (E) for a polyhedron is described by the formula $F + V - E = 2$.

Try the following questions.

1. How many diagonals does a dodecagon (twelve-sided polygon) have?
2. What is the angle sum for a dodecagon?
3. Both ends of a prism are dodecagons. How many diagonals does it have?
4. Show that $F + V - E = 2$ applies to the prism in question 3.



For solutions to Extra Help, turn to Appendix A, Topic 4.



Extensions

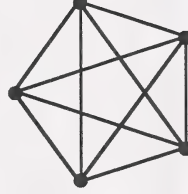
There is a similar problem to the one of determining the number of diagonals. In a group of n persons, how many different handshakes occur if every person shakes the hand of every other person. You can use a model to determine the answer. Let dots represent individuals and connecting line segments represent handshakes.



These two dots can be connected with just one segment.



This can be interpreted to mean that for a group of two persons, there will be one handshake. In a group of five persons, there will be ten handshakes.



Try the following questions.

1. Determine the number of handshakes in groups of different sizes. Tabulate your results and give a formula to express the general relationship between handshakes and group size.

Use the chart below to tabulate your results.

Group Size	Number of Different Handshakes
2	
3	
4	
5	
6	
7	
8	
9	
10	

2. Use your general expression for handshakes to complete the following statement.

The number of diagonals in an n -sided polygon is [your expression for handshakes] $- n$.

3. Show that the statement in question 2 is equivalent to the number of diagonals in an n -sided polygon $\frac{1}{2}n(n-3)$.
4. Why do you subtract n from your general expression for handshakes in order to get an expression for diagonals?



For solutions to Extensions,
turn to **Appendix A, Topic 4**.

Unit Summary



What You Have Learned

You have learned the meaning of perimeter, area, and volume. You have also learned how to quantify each of these for a variety of plane figures and solids. You also discovered many relationships involving vertices, edges, sides, faces, and diagonals. You should now be better able to solve practical problems and have a better appreciation for geometry.

You are now ready to
complete the **Unit Assignment**.

Appendices



Appendix A Solutions

Review

Topic 1 Perimeter

Topic 2 Area

Topic 3 Volume

Topic 4 Relationships for Geometric
Figures



Appendix B Nets

Cylinder

Cone

Pyramid

Rectangular Prism

Triangular Prism

Regular Tetrahedron

Regular Hexahedron

Regular Octahedron

Regular Dodecahedron

Regular Icosahedron

Square Prism

Square Pyramid



Appendix A Solutions



Review

1. a. Enter 85



Press

Display reads

9.219544457

$\sqrt{85} \approx 9.22$

b. Enter 96



Press

Display reads

9.797958971

$\sqrt{96} \approx 9.80$

c. Enter 30



Press

Display reads

3.107232506

$\sqrt[3]{30} \approx 3.11$

d. Enter 941



Press

Display reads

9.799333566

$\sqrt[3]{941} \approx 9.80$

2. a. $(x + 3)(x - 21)$

b. $(x - 4)(x + 5)$

If you had trouble with question 1, go to Mathematics 23, Unit 1, Powers and Radicals.

If you had trouble with question 2, go to Mathematics 23, Unit 2, Algebra.



Exploring Topic 1

Activity 1

Determine the perimeter of a polygon.

1. a. $P = a + b + c$

$$= 105 \text{ cm} + 42 \text{ cm} + 132 \text{ cm}$$

$$= 279 \text{ cm}$$

The perimeter is 279 cm.

b. $P = 2(l + w)$

$$= 2(35 \text{ cm} + 24 \text{ cm})$$

$$= 2(59 \text{ cm})$$

$$= 118 \text{ cm}$$

The perimeter is 118 cm.

c. $P = 77 \text{ km} + 187 \text{ km} + 253 \text{ km} + 209 \text{ km} + 297 \text{ km} + 275 \text{ km}$
 $+ 196 \text{ km}$

$$= 1494 \text{ km}$$

The perimeter is 1494 km.

2. a. $P = a + b + c$
 $= 3 \text{ cm} + 3 \text{ cm} + 4 \text{ cm}$
 $= 10 \text{ cm}$
 The perimeter is 10 cm.
- b. $P = 2(l + w)$
 $= 2(610 \text{ m} + 260 \text{ m})$
 $= 2(870 \text{ m})$
 $= 1740 \text{ m}$
 The perimeter is 1740 m.

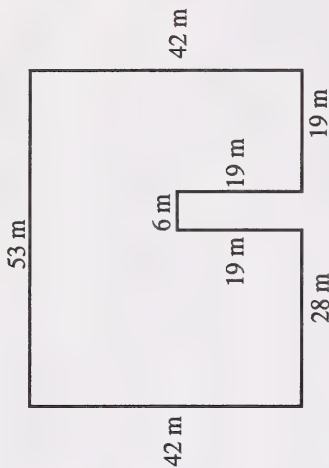
- c. $P = 10 \text{ m} + 29 \text{ m} + 21 \text{ m} + 39 \text{ m} + 23 \text{ m}$
 $= 122 \text{ m}$
 The perimeter is 122 m.

3. Let $z = 5.9$ and $y = 5.1$.
 Let x cm be the length of the remaining side.

$$\begin{aligned}\text{Then } x^2 + y^2 &= z^2 \\ x^2 &= z^2 - y^2 \\ &= 5.9^2 - 5.1^2 \\ &= 34.81 - 26.01 \\ &= 8.80 \\ x &= \sqrt{8.80} \\ &= 3.0 \text{ cm}\end{aligned}$$

- $P = 3.0 \text{ cm} + 5.1 \text{ cm} + 5.9 \text{ cm}$
 $= 14.0 \text{ cm}$
 The perimeter of the right triangle is 14.0 cm.

4. First determine the length of the unknown sides and place the values on the diagram.

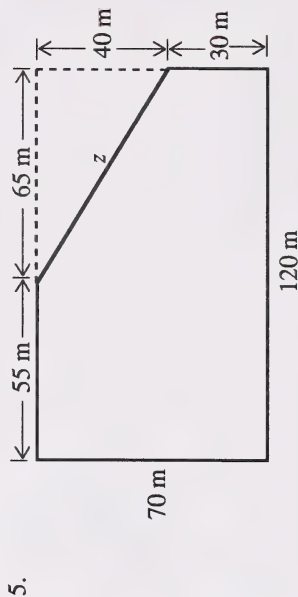


Then determine the perimeter as you would for a polygon.

$$P = 53 \text{ m} + 42 \text{ m} + 19 \text{ m} + 19 \text{ m} + 6 \text{ m} + 19 \text{ m} + 28 \text{ m} + 42 \text{ m}$$

$$= 228 \text{ m}$$

The perimeter is 228 m.



$$65^2 + 40^2 = z^2$$

$$z^2 = 4225 + 1600$$

$$= 5825$$

$$z = \sqrt{5825}$$

$$\approx 76$$

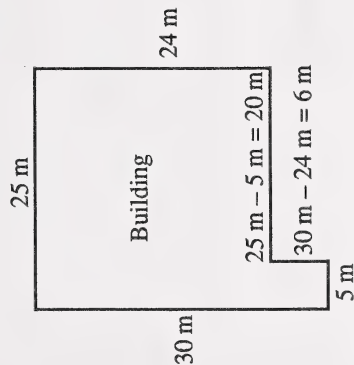
The length of the slanted side is 76 m.

$$P = 55 \text{ m} + 70 \text{ m} + 120 \text{ m} + 30 \text{ m} + 76 \text{ m}$$

$$= 351 \text{ m}$$

The perimeter is 351 m.

6.

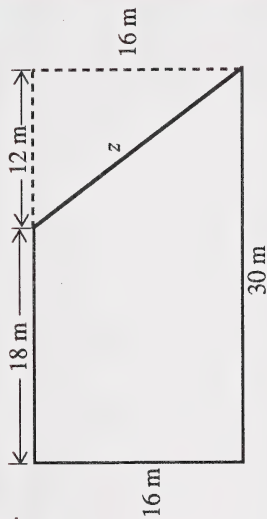


$$P = 25 \text{ m} + 24 \text{ m} + 20 \text{ m} + 6 \text{ m} + 5 \text{ m} + 30 \text{ m}$$

$$= 110 \text{ m}$$

The perimeter of the building is 110 m.

7.



$$12^2 + 16^2 = z^2$$

$$z^2 = 144 + 256$$

$$= 400$$

$$z = \sqrt{400}$$

$$= 20$$

$$P = 18 \text{ m} + 20 \text{ m} + 30 \text{ m} + 16 \text{ m}$$

$$= 84 \text{ m}$$

There are 2 plants for every metre.

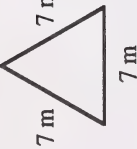
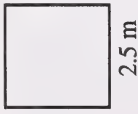
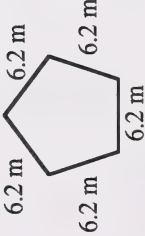
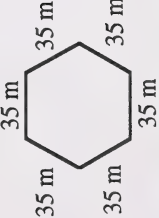
The number of plants is $84 \times 2 = 168$ plants.

There will be 168 cotoneaster plants needed for the hedge around the lot.

Activity 2

Develop and use the formula for the perimeter of a regular polygon.

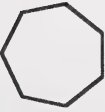
1.

Figure (not all drawn to the same scale)	Classification	Number of Sides	Side Length (m)	Perimeter (m)
 7 m 7 m 7 m	Equilateral Triangle	3	7 m	$7\text{ m} + 7\text{ m} + 7\text{ m} = 21\text{ m}$
 2.5 m 2.5 m 2.5 m 2.5 m	Square	4	2.5 m	$2.5\text{ m} + 2.5\text{ m} + 2.5\text{ m} + 2.5\text{ m} = 10.0\text{ m}$
 6.2 m 6.2 m 6.2 m 6.2 m 6.2 m	Regular Pentagon	5	6.2 m	$6.2\text{ m} + 6.2\text{ m} + 6.2\text{ m} + 6.2\text{ m} + 6.2\text{ m} = 31.0\text{ m}$
 35 m 35 m 35 m 35 m 35 m 35 m	Regular Hexagon	6	35 m	$35\text{ m} + 35\text{ m} + 35\text{ m} + 35\text{ m} + 35\text{ m} + 35\text{ m} = 210\text{ m}$

2. For the equilateral triangle, the perimeter is 3 times the length of a side.
 For the square, the perimeter is 4 times the length of a side.
 For the regular pentagon, the perimeter is 5 times the length of a side.
 For the regular hexagon, the perimeter is 6 times the length of a side.

3. $P = ns$

4.

Figure (not drawn to scale)	Classification	Number of Sides	Side Length (m)	Perimeter (m)
	Regular Heptagon	7	21 m	$P = 7(21 \text{ m})$ $= 147 \text{ m}$
	Regular Octagon	8	1.25 m	$P = 8(1.25 \text{ m})$ $= 10.00 \text{ m}$
	Regular Nonagon	9	12 m	$P = 9(12 \text{ m})$ $= 108 \text{ m}$
	Regular Decagon	10	$s = \frac{P}{n}$ $= \frac{120 \text{ m}}{10}$ $= 12 \text{ m}$	120 m

$$5. P = ns$$

$$= 12(10.5 \text{ cm})$$

$$= 126$$

The perimeter of the dodecagon is 126 cm.

$$6. P = ns$$

$$n = \frac{P}{s}$$

$$= \frac{360 \text{ cm}}{12 \text{ cm}}$$

$$= 30$$

The polygon has 30 sides.

$$7. P = ns$$

$$n = \frac{P}{s}$$

$$= \frac{385 \text{ cm}}{35 \text{ cm}}$$

$$= 11$$

The polygon has 11 sides.

Activity 3

Determine and use the fact that π is the ratio between the circumference and diameter of a circle.

1. Sample entries are given.

Answers will vary. Answers in the last column should be similar.

Object	Circumference (C)	Diameter (d)	$C \div d$
bicycle wheel	219.9 cm	70.0 cm	3.14
soup can	26.7 cm	8.5 cm	3.14
dinner plate	88.0 cm	28.0 cm	3.14
lid	40.5 cm	12.9 cm	3.14
roll of tape	28.6 cm	9.1 cm	3.14

2.

Object	Circumference (C)	Diameter (d)	$C \div d$
trailer wheel	140.0 cm	44.5 cm	3.15
soup can	23.2 cm	7.4 cm	3.14
fruit can	31.4 cm	10.0 cm	3.14
vegetable can	45.6 cm	14.5 cm	3.14
plate	73.8 cm	23.6 cm	3.13
saucer	40.0 cm	12.7 cm	3.15

Note: The variation in the value of the ratio is due to measuring error.

3. The circumference is approximately 3.14 times as large as the diameter of the circle.

$$\begin{aligned}
 4. \quad C &= \pi d \\
 &= (\pi)(77 \text{ cm}) \\
 &= 241.90 \text{ cm} \\
 &\approx 2.42 \text{ m}
 \end{aligned}$$

The number of turns in 1.5 km is $\frac{1500 \text{ m}}{2.42 \text{ m}} \approx 620$.

Each wheel must make 620 turns as Sandy pedals to school.

5. $r = 2.10 \text{ cm}$

$$C = 2\pi r$$

$$= 2(\pi)(2.10 \text{ cm})$$

$$\approx 13.19 \text{ cm}$$

The circumference is 13.19 cm.

6. The width of the plank is equal to the diameter of the tree.

$$C = \pi d$$

$$d = \frac{C}{\pi}$$

$$= \frac{7.5 \text{ m}}{\pi}$$

$$\approx 2.4 \text{ m}$$

The maximum plank width is 2.4 m.

7. $C = \pi d$

$$= (\pi)(5.6 \text{ cm})$$

$$\approx 17.6 \text{ cm}$$

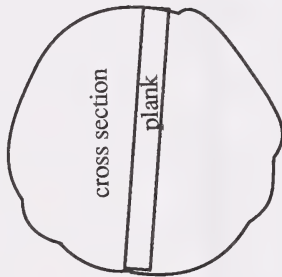
The circumference of the stain was 17.6 cm.

8. $C = 2\pi r$

$$= 2(\pi)(15 \text{ cm})$$

$$\approx 94.2 \text{ cm}$$

When the larger wheel makes one complete rotation, 94.2 cm of belt passes over the smaller wheel.



$$C = 2\pi r$$

$$= 2(\pi)(7.5 \text{ cm})$$

$$\approx 47.1 \text{ cm}$$

The smaller wheel makes one complete rotation for every 47.1 cm of belt that passes.

The smaller wheel makes $\left(\frac{94.2 \text{ cm}}{47.1 \text{ cm}}\right)$ turns $= 2$ turns.

The smaller wheel will make two complete turns for every one turn of the larger wheel.

Activity 4

Solve problems involving the perimeter of closed figures.

1. The curved parts have a length equal to a circle of radius 10 cm.

$$C = 2\pi r$$

$$= 2(\pi)(10 \text{ cm})$$

$$\approx 62.8 \text{ cm}$$

Each of the straight parts of the curve have a length of 20 cm.

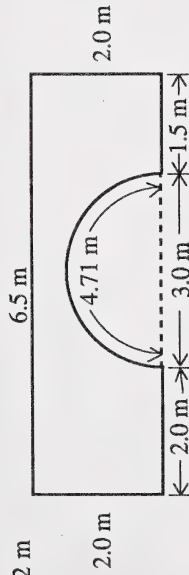
$$P = 62.8 \text{ cm} + 20 \text{ cm} + 20 \text{ cm}$$

$$= 102.8 \text{ cm}$$

The perimeter of the figure is 102.8 cm.

2. The curved part is a semicircle of diameter 3.0 m.

$$\begin{aligned}C &= \pi d \\&= (\pi)(3.0 \text{ m}) \\&\approx 9.42 \text{ m}\end{aligned}$$



The length of the curved part is $\frac{1}{2}C = \frac{9.42}{2} \text{ m} = 4.71 \text{ m}$.

$$\begin{aligned}P &= 4.71 \text{ m} + 1.5 \text{ m} + 2.0 \text{ m} + 6.5 \text{ m} + 2.0 \text{ m} + 2.0 \text{ m} \\&\approx 18.7 \text{ m}\end{aligned}$$

The perimeter of the figure is 18.7 m.

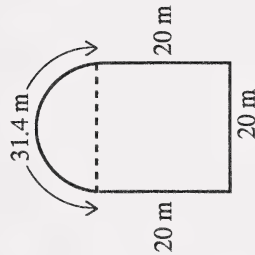
3. The semicircle is of diameter 20 m.

$$\begin{aligned}C &= \pi d \\&= (\pi)(20 \text{ m}) \\&\approx 62.8 \text{ m}\end{aligned}$$

The length of the semicircle is $\frac{62.8 \text{ m}}{2} = 31.4 \text{ m}$.

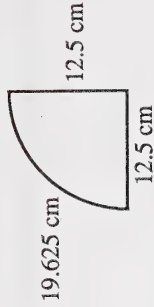
$$\begin{aligned}P &= 20 \text{ m} + 20 \text{ m} + 20 \text{ m} + 31.4 \text{ m} \\&\approx 91 \text{ m}\end{aligned}$$

The distance around the lot is 91 m.



4. $C = \pi d$

$$\begin{aligned}&= (\pi)(25 \text{ cm}) \\&\approx 78.5 \text{ cm}\end{aligned}$$



The length of the curved edge is $\frac{78.5 \text{ cm}}{4} = 19.625 \text{ cm}$.

The length of each of the straight sides is equal to the radius
 $= \frac{25 \text{ cm}}{2} = 12.5 \text{ cm}$.

$$\begin{aligned}P &= 12.5 \text{ cm} + 12.5 \text{ cm} + 19.625 \text{ cm} \\&\approx 44.6 \text{ cm}\end{aligned}$$

The perimeter of each piece of pizza is 44.6 cm.

5. The curved parts form a semicircle.

The length of the parts $= \frac{1}{2}C$

$$\begin{aligned}&= \frac{1}{2}(2\pi r) \\&= \pi r \\&= (\pi)(50 \text{ m}) \\&\approx 157 \text{ m}.\end{aligned}$$

The length of the straight section between the curved parts can be seen to have a length of 150 m.

$$\begin{aligned}P &= 150 \text{ m} + 157 \text{ m} + 100 \text{ m} + 100 \text{ m} + 50 \text{ m} + 50 \text{ m} \\&\quad + 50 \text{ m} + 100 \text{ m} + 100 \text{ m} \\&= 857 \text{ m}\end{aligned}$$

The perimeter of the public building is 857 m.

Extra Help

1. $P = a + b + c$
 $= 3.8 \text{ m} + 1.4 \text{ m} + 4.9 \text{ m}$
 $= 10.1 \text{ m}$

The perimeter of the triangle is 10.1 m.

2. $C = \pi d$
 $= (\pi)(2.85 \text{ cm})$
 $\approx 8.95 \text{ cm}$

The perimeter of the circle is 8.95 cm.

3. $P = 2(l + w)$
 $= 2(30 \text{ m} + 16 \text{ m})$
 $= 2(46 \text{ m})$
 $= 92 \text{ m}$

The perimeter of the garden is 92 m.

4. $P = ns$
 $s = \frac{P}{n}$
 $= \frac{125 \text{ cm}}{5}$
 $= 25 \text{ cm}$

The length of each side is 25 cm.

Extensions

1. $P = \sin\left(\frac{180^\circ}{n}\right) 2nr$
 $= \sin\left(\frac{180^\circ}{9}\right) (2)(9)(8.00 \text{ cm})$
 $= \sin(20^\circ)(18)(8.00 \text{ cm})$

Enter	Display
C	0
20	20
sin	0.342020143
×	0.342020143
18	18
×	6.15636258
8	8
=	49.25090064

The perimeter of the nanogon is 49.3 cm.

$$2. P = \sin\left(\frac{180^\circ}{n}\right) 2nr$$

$$= \sin\left(\frac{180^\circ}{7}\right) (2)(7)(10.0 \text{ cm})$$

Enter	Display
C	0
180	180
÷	180
7	7
=	25.71428571
sin	0.433883739
×	0.433883739
2	2
×	0.867767478
7	7
×	6.074372348
10	10
=	60.74372348

The perimeter of the heptagon is 60.7 cm.

$$3. P = \sin\left(\frac{180^\circ}{8}\right) 2nr$$

$$= \sin\left(\frac{180^\circ}{8}\right) (2)(8)(7.50 \text{ m})$$

$$= \sin(22.5^\circ) 120$$

Enter	Display
C	0
22.5	22.5
sin	0.382683432
×	0.382683432
120	120
=	45.92201189

The perimeter of the octagon is 45.9 m.



Exploring Topic 2

Activity 1

Determine the area of a polygon.

1. $l = 1.5 \text{ m}$
 $= 150 \text{ cm}$
 $w = 0.50 \text{ m}$
 $= 50 \text{ cm}$
 $A = lw$
 $= 150 \text{ cm} \times 50 \text{ cm}$
 $= 7500 \text{ cm}^2$

The area of the rectangle is 7500 cm^2 .

2. $l = w$
 $= 1 \text{ m}$
 $= 100 \text{ cm}$
 $A = lw$
 $= 100 \text{ cm} \times 100 \text{ cm}$
 $= 10\,000 \text{ cm}^2$

There are $10\,000 \text{ cm}^2$ in a square metre.

3. $A = lw$
 $= 8.4 \text{ m} \times 3.2 \text{ m}$
 $= 26.88 \text{ m}^2$

To paint 26.88 m^2 , 3 L of paint is needed. The cost will be $\$9.45 \times 3 = \$28.35$.

The paint will cost Margaret \$28.35.

4. $A = bh$
 $= 68 \text{ cm} \times 9.5 \text{ cm}$
 $= 646 \text{ cm}^2$

The area is 646 cm^2 .

5. $A = bh$
 $= 2.2 \text{ m} \times 1.9 \text{ m}$
 $= 4.18 \text{ m}^2$

The area of the parallelogram is 4.2 m^2 .

6. $A = bh$
 $= 740 \text{ cm} \times 310 \text{ cm}$ or $7.40 \text{ m} \times 3.10 \text{ m}$
 $= 229\,400 \text{ cm}^2$ or 22.94 m^2

An area of $229\,400 \text{ cm}^2$ or 22.9 m^2 must be painted.

$$7. \quad b = 18 \text{ cm}$$

$$h = 6 \text{ cm}$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(18 \text{ cm})(6 \text{ cm})$$

$$= 54 \text{ cm}^2$$

The area of the triangle is 54 cm^2 .

$$8. \quad A = \frac{1}{2}bh$$

$$= \frac{1}{2}(8.0 \text{ m})(9.6 \text{ m})$$

$$= 38.4 \text{ m}^2$$

The area of the triangle is 38.4 m^2 .

$$9. \quad A = \frac{1}{2}bh$$

$$= \frac{1}{2}(19 \text{ m})(12 \text{ m})$$

$$= 114 \text{ m}^2$$

The area of the gable is 114 m^2 .

$$10. \quad A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(2.0 \text{ cm})(5.1 \text{ cm} + 7.1 \text{ cm})$$

$$= (1.0 \text{ cm})(12.2 \text{ cm})$$

$$= 12.2 \text{ cm}^2$$

The area of the trapezoid is 12.2 cm^2 .

$$11. \quad A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(19 \text{ m})(88 \text{ m} + 61 \text{ m})$$

$$= \frac{1}{2}(19 \text{ m})(149 \text{ m})$$

$$= 1416 \text{ m}^2$$

The area of the garden is 1416 m^2 .

$$12. \quad A = \frac{1}{2}h(a+b)$$

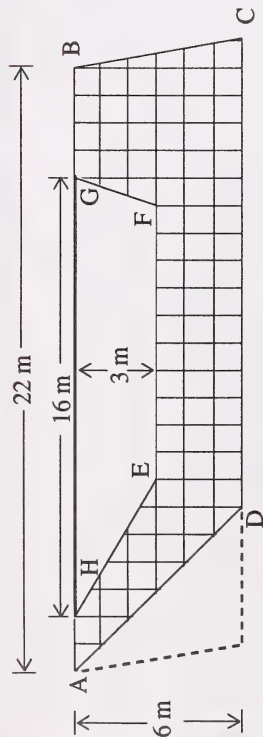
$$= \frac{1}{2}(60 \text{ cm})(80 \text{ cm} + 150 \text{ cm})$$

$$= (30 \text{ cm})(230 \text{ cm})$$

$$= 6900 \text{ cm}^2$$

The area of the table is 6900 cm^2 .

13.



Area of trapezoid ABCD is

$$\begin{aligned}
 & \frac{1}{2}h(a+b) \\
 &= \frac{1}{2}(6\text{ m})(22\text{ m} + 16\text{ m}) \\
 &= (3\text{ m})(39\text{ m}) \\
 &= 117\text{ m}^2.
 \end{aligned}$$

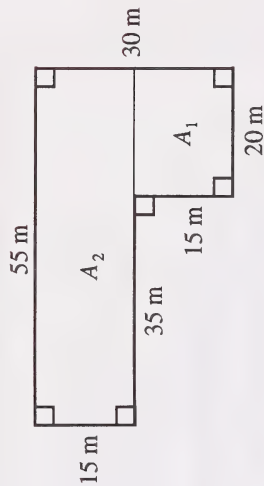
Area of trapezoid EFGH is

$$\begin{aligned}
 & \frac{1}{2}h(a+b) \\
 &= \frac{1}{2}(3\text{ m})(16\text{ m} + 10\text{ m}) \\
 &= 39\text{ m}^2.
 \end{aligned}$$

The area of the region covered by the grid is
 $117\text{ m}^2 - 39\text{ m}^2 = 78\text{ m}^2$.

The area covered by the grid is 78 m^2 .

14.

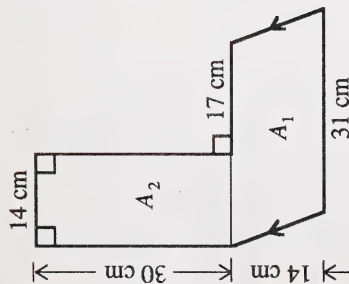


$$\begin{aligned}
 A_1 &= hw \\
 &= 15\text{ m} \times 20\text{ m} \\
 &= 300\text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 A_2 &= hw \\
 &= 35\text{ m} \times 30\text{ m} \\
 &= 1050\text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 A &= A_1 + A_2 \\
 &= 300\text{ m}^2 + 1050\text{ m}^2 \\
 &= 1350\text{ m}^2
 \end{aligned}$$

15.



$$A_1 = bh$$

$$= (31 \text{ cm})(14 \text{ cm})$$

$$= 434 \text{ cm}^2$$

$$A_2 = lw$$

$$= (30 \text{ cm})(14 \text{ cm})$$

$$= 420 \text{ cm}^2$$

$$A = A_1 + A_2$$

$$= 434 \text{ cm}^2 + 420 \text{ cm}^2$$

$$= 854 \text{ cm}^2$$

The letter L would be made up of 854 cm^2 of cardboard.

Activity 2

Develop and use a formula for the area of a regular polygon.

$$1. \quad A = \frac{1}{2}bh$$

$$= \frac{1}{2}sa$$

$$= \frac{1}{2}as \quad (\text{Variables in a formula are often put in alphabetic order.})$$

2. Ten triangles fit in the decagon.

3. The area of the decagon is ten times larger than the area of the triangle.

$$4. \quad \text{For the decagon, } A = 10\left(\frac{1}{2}as\right)$$

$$= 5as$$

5. For any regular n -sided polygon,

$$A = \frac{1}{2}ans.$$

$$6. \quad A = \frac{1}{2}ans$$

$$= \frac{1}{2}(5.7 \text{ cm})(7)(5.5 \text{ cm})$$

$$\approx 110 \text{ cm}^2$$

The area of the polygon is 110 cm^2 .

$$7. \quad P = ns$$

$$s = \frac{P}{n} \\ = \frac{9.7 \text{ m}}{10} \\ = 0.97 \text{ m}$$

$$A = \frac{1}{2} \text{ ans} \\ = \frac{1}{2}(1.5 \text{ m})(10)(0.97 \text{ m}) \\ = \frac{1}{2}(1.5 \text{ m})(9.7 \text{ m}) \\ = 7.275 \text{ m}^2$$

The area of the polygon is 7.3 m^2 .

$$8. \quad A = s^2 \qquad \text{or} \qquad A = \frac{1}{2} \text{ ans} \\ = (24 \text{ cm})^2 \\ = 576 \text{ cm}^2$$

The area of the square is 576 cm^2 .

$$9. \quad A = \frac{\sqrt{3}}{4} s^2 \\ = \frac{\sqrt{3}}{4} \left(\frac{42 \text{ m}}{3} \right)^2 \\ = \frac{\sqrt{3}}{4} (14 \text{ m})^2 \\ \doteq (0.433)(196 \text{ m}^2) \\ \doteq 85 \text{ m}^2$$

The area of the triangle is 85 m^2 .

$$10. \quad A = \frac{1}{2} \text{ ans} \\ = \frac{1}{2} \left(\frac{18 \text{ m}}{2} \right) (8)(10 \text{ m}) \\ = (9 \text{ m})(4)(10 \text{ m}) \\ = 360 \text{ m}^2$$

The area of the roof is 360 m^2 .

$$11. \quad A = \frac{\sqrt{3}}{4} s^2 \\ = \frac{\sqrt{3}}{4} (12 \text{ cm})^2 \\ \doteq (0.433)(144 \text{ cm}^2) \\ \doteq 62 \text{ cm}^2$$

The area of the triangle is 62 cm^2 .

12. $A = \frac{1}{2}as$

$$= \frac{1}{2}(60 \text{ cm})(16)(24 \text{ cm})$$

$$= 11\,520 \text{ cm}^2$$

The area of the polygon is $11\,520 \text{ cm}^2$.

13. $A = \frac{1}{2}as$

$$= \frac{1}{2}(220 \text{ m})(5)\left(\frac{1600 \text{ m}}{5}\right)$$

$$= \frac{1}{2}(220 \text{ m})(1600 \text{ m})$$

$$= 176\,000 \text{ m}^2$$

The area occupied by the Pentagon Building is $176\,000 \text{ m}^2$.

14. $A = s^2$

$$s = \sqrt{A}$$

$$= \sqrt{45 \text{ cm}^2}$$

$$\doteq 6.7 \text{ cm}$$

The length of each side of the square is 6.7 cm .

Activity 3

Solve problems involving the area of circles.

1. $r = \frac{6 \text{ cm}}{2}$

$$= 3 \text{ cm}$$

$$A = \pi r^2$$

$$= (\pi)(3 \text{ cm})^2$$

$$= (\pi)(9 \text{ cm}^2)$$

$$\doteq 28 \text{ cm}^2$$

The area of the circle is 28 cm^2 .

2. $r = 26.0 \text{ cm}$

$$A = \pi r^2$$

$$= (\pi)(26.0 \text{ cm})^2$$

$$\doteq 2124 \text{ cm}^2$$

The area of the circle is 2124 cm^2 .

3. $d = 43.6 \text{ cm}$

$r = 21.8 \text{ cm}$

$A = \pi r^2$

$= (\pi)(21.8 \text{ cm})^2$

$\approx 1493 \text{ cm}^2$

The area of the circle is 1493 cm^2 .

4. $r = 1.00 \text{ m}$

$A = \pi r^2$

$= (\pi)(1.00 \text{ m})^2$

$\approx 3.14 \text{ m}^2$

The area of the circle is 3.14 m^2 .

5. $r = 17.8 \text{ cm}$

$A = \pi r^2$

$= (\pi)(17.8 \text{ cm})^2$

$\approx 995 \text{ cm}^2$

The area of the circle is 995 cm^2 .

6. $d = 3.3 \text{ cm}$

$r = 1.65 \text{ cm}$

$A = \pi r^2$

$= (\pi)(1.65 \text{ cm})^2$

$\approx 8.6 \text{ cm}^2$

The area of the clock face is 8.6 cm^2 .

7. $d = 22.5$

$r = 11.25$

$A = \pi r^2$

$A = (\pi)(11.25 \text{ cm})^2$

$\approx 398 \text{ cm}^2$

A layer of dough having an area of 398 cm^2 is needed to make the top crust.

8. For the 18 cm pizza,

$r = 9.0 \text{ cm}$.

$A = \pi r^2$

$= (\pi)(9.0 \text{ cm})^2$

$\approx 254 \text{ cm}^2$

Cost is $\frac{549\phi}{254 \text{ cm}^2} = 2.2\phi / \text{cm}^2$.

For the 26 cm pizza,

$$r = 13 \text{ cm.}$$

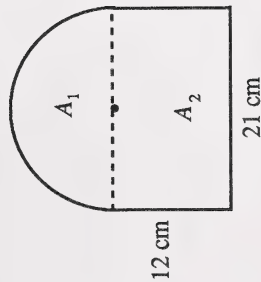
$$A = \pi r^2$$

$$= (\pi)(13 \text{ cm})^2$$

$$\approx 531 \text{ cm}^2$$

$$\text{Cost is } \frac{825\phi}{531 \text{ cm}^2} = 1.6\phi / \text{cm}^2.$$

The larger pizza is the better buy. Its cost per square centimetre is much less.



$$A_1 = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} (\pi) \left(\frac{21 \text{ cm}}{2} \right)^2$$

$$\approx 173 \text{ cm}^2$$

$$A_2 = lw$$

$$= (21 \text{ cm})(12 \text{ cm})$$

$$= 252 \text{ cm}^2$$

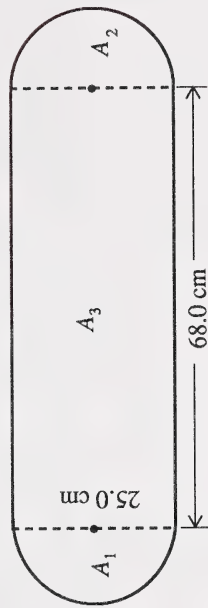
$$A = A_1 + A_2$$

$$= 173 \text{ cm}^2 + 252 \text{ cm}^2$$

$$= 425 \text{ cm}^2$$

The area of the figure is 425 cm^2 .

10.



$$A_1 + A_2 = \pi r^2$$

$$= (\pi) \left(\frac{25.0 \text{ cm}}{2} \right)^2$$

$$\approx 491 \text{ cm}^2$$

$$A_3 = lw$$

$$= (68.0 \text{ cm})(25.0 \text{ cm})$$

$$= 1700 \text{ cm}^2$$

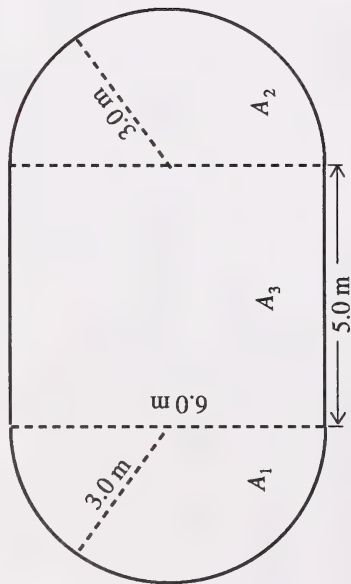
$$A = (A_1 + A_2) + A_3$$

$$= 491 \text{ cm}^2 + 1700 \text{ cm}^2$$

$$= 2191 \text{ cm}^2$$

The area of the figure is 2191 cm^2 .

11. Bessie has access to the region of grass shown.



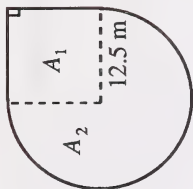
$$\begin{aligned} A_1 + A_2 &= \pi r^2 \\ &= (\pi)(3.0 \text{ m})^2 \\ &= 28.27 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A_3 &= lw \\ &= (6.0 \text{ m})(5.0 \text{ m}) \\ &= 30.0 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= (A_1 + A_2) + A_3 \\ &= 28.27 \text{ m}^2 + 30.0 \text{ m}^2 \\ &\approx 58 \text{ m}^2 \end{aligned}$$

Bessie has access to 58 m^2 of grass.

- 12.



$$\begin{aligned} A_1 &= lw \\ &= (12.5 \text{ m})(12.5 \text{ m}) \\ &\approx 156 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A_2 &= \frac{3}{4}(\pi r^2) \\ &= \frac{3}{4}(\pi)(12.5 \text{ m})^2 \\ &\approx 368 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= A_1 + A_2 \\ &= 156 \text{ m}^2 + 368 \text{ m}^2 \\ &= 524 \text{ m}^2 \\ &\approx 524 \text{ m}^2 \end{aligned}$$

The area of the figure is 524 m^2 .

13.

	Figure A	Figure B	Figure C	Figure D
a. Perimeter	$P = 2(l + w)$ $= 2(5.00 \text{ m} + 3.00 \text{ m})$ $= 16.00 \text{ m}$	$P = 4s$ $= 4(4.00 \text{ m})$ $= 16.00 \text{ m}$	$C = 2\pi r$ $= 2(\pi)(2.54 \text{ m})$ $\approx 15.96 \text{ m}$	$P = 2\pi r + 2.00 \text{ m} + 2.00 \text{ m}$ $= 2(\pi)(1.91 \text{ m}) + 4.00 \text{ m}$ $\approx 16.00 \text{ m}$
b. Area	$A = lw$ $= (5.00 \text{ m})(3.00 \text{ m})$ $= 15.0 \text{ m}^2$	$A = s^2$ $= (4.00 \text{ m})^2$ $= 16.0 \text{ m}^2$	$A = \pi r^2$ $= (\pi)(2.54 \text{ m})^2$ $\approx 20.3 \text{ m}^2$	$A = \pi r^2 + lw$ $= (\pi)(1.91 \text{ m})^2 + [2(1.91 \text{ m})](2.00 \text{ m})$ $\approx 11.46 \text{ m}^2 + 7.64 \text{ m}^2$ $\approx 19.1 \text{ m}^2$

c. The circle has the greatest area.

14.

	a. Perimeter	b. Area
Figure A	$P = \frac{1}{2}(2\pi r) + 2(5.00 \text{ cm}) + 2(2.72 \text{ cm})$ $= (\pi)(2.72 \text{ cm}) + 10.00 \text{ cm} + 5.44 \text{ cm}$ $\approx 8.55 \text{ cm} + 10.00 \text{ cm} + 5.44 \text{ cm}$ $\approx 23.99 \text{ cm}$	$A = \frac{1}{2}(\pi r^2) + lw$ $= \frac{1}{2}(\pi)(2.72 \text{ cm})^2 + (5.00 \text{ cm})(2.72 \text{ cm})$ $\approx 11.62 \text{ cm}^2 + 13.60 \text{ cm}^2$ $\approx 25.2 \text{ cm}^2$
Figure B	$P = \frac{1}{2}(2\pi r) + 9.34 \text{ cm}$ $= (\pi)\left(\frac{9.34 \text{ cm}}{2}\right) + 9.34 \text{ cm}$ $\approx 24.01 \text{ cm}$	$A = \frac{1}{2}\pi r^2$ $= \frac{1}{2}(\pi)\left(\frac{9.34 \text{ cm}}{2}\right)^2$ $\approx 34.3 \text{ cm}^2$
Figure C	$P = 2(l + w)$ $= 2(8.00 \text{ cm} + 4.00 \text{ cm})$ $= 2(12.00 \text{ cm})$ $= 24.00 \text{ cm}$	$A = lw$ $= (8.00 \text{ cm})(4.00 \text{ cm})$ $= 32.0 \text{ cm}^2$
Figure D	$C = 2\pi r$ $= 2(\pi)(3.82 \text{ cm})$ $\approx 24.00 \text{ cm}$	$A = \pi r^2$ $= (\pi)(3.82 \text{ cm})^2$ $\approx 45.8 \text{ cm}^2$

c. The circle has the largest area.

15. a. For the triangle, Figure A,

$$A = \frac{\sqrt{3}}{4} s^2$$

$$= \frac{\sqrt{3}}{4} (1334 \text{ m})^2$$

$$\approx 770\,570 \text{ m}^2$$

The area of the triangle is $770\,570 \text{ m}^2$.

For the circle, Figure B,

$$A = \pi r^2$$

$$= (\pi)(637 \text{ m})^2$$

$$\approx 1\,274\,760 \text{ m}^2$$

The area of the circle is $1\,274\,760 \text{ m}^2$.

For the square, Figure C,

$$A = s^2$$

$$= (1000 \text{ m})^2$$

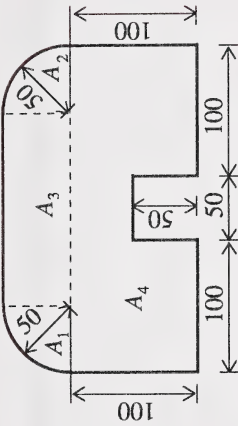
$$= 1\,000\,000 \text{ m}^2$$

The area of the square is $1\,000\,000 \text{ m}^2$.

The circle encloses the greatest area.

- b. Your land should be in the shape of a circle for the greatest area.

16. Of all the figures with a given perimeter, the circle has the greatest area.



17.

$$A_1 + A_2 = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} (\pi) (50.0 \text{ m})^2$$

$$= 3927 \text{ m}^2$$

$$A_3 = lw$$

$$= (150 \text{ m})(50 \text{ m})$$

$$= 7500 \text{ m}^2$$

$$A_4 = lw - (50 \text{ m})(50 \text{ m})$$

$$= (250 \text{ m})(100 \text{ m}) - 2500 \text{ m}^2$$

$$= 25\,000 \text{ m}^2 - 2500 \text{ m}^2$$

$$= 22\,500 \text{ m}^2$$

$$A = (A_1 + A_2) + A_3 + A_4$$

$$= 3927 \text{ m}^2 + 7500 \text{ m}^2 + 22\,500 \text{ m}^2$$

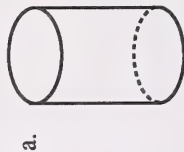
$$= 33\,927 \text{ m}^2$$

The public building occupies $33\,927 \text{ m}^2$ of land.

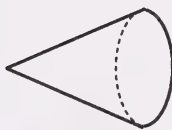
Activity 4

Calculate the surface area of solids.

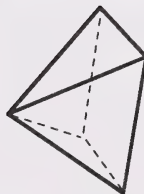
1. Your solids should look like the following.



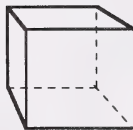
b.



c.



d.



e.



2. For each of the front and back,

$$A = lw$$

$$= (4.50 \text{ m})(2.00 \text{ m})$$

$$= 9.00 \text{ m}^2$$

For each of the left and right sides,

$$A = (4.50 \text{ m})(1.25 \text{ m})$$

$$= 5.63 \text{ m}^2$$

For each of the top and bottom,

$$A = (2.00 \text{ m})(1.25 \text{ m})$$

$$= 2.50 \text{ m}^2$$

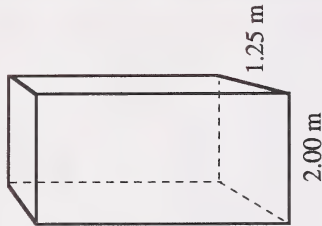
For the whole prism,

$$A = 2(9.00 \text{ m}^2) + 2(5.63 \text{ m}^2) + 2(2.50 \text{ m}^2)$$

$$= 18.00 \text{ m}^2 + 11.26 \text{ m}^2 + 5.00 \text{ m}^2$$

$$= 34.26 \text{ m}^2$$

The surface area of the prism is 34.26 m^2 .



3. $A = 2(28 \text{ cm})(12 \text{ cm}) + 2(28 \text{ cm})(6.5 \text{ cm}) + 2(12 \text{ cm})(6.5 \text{ cm})$

$$= 672 \text{ cm}^2 + 364 \text{ cm}^2 + 156 \text{ cm}^2$$

$$= 1192 \text{ cm}^2$$

The surface area of the cereal box is 1192 cm^2 .

4. For each of the top and bottom,

$$A = \frac{1}{2}as$$

$$= \frac{1}{2}(8.0 \text{ cm})(5)(22 \text{ cm})$$

$$= 440 \text{ cm}^2$$

For each of the vertical sides,

$$A = lw$$

$$= (22 \text{ cm})(6.0 \text{ cm})$$

$$= 132 \text{ cm}^2$$

For the box,

$$A = 2(440 \text{ cm}^2) + 5(132 \text{ cm}^2)$$

$$= 1540 \text{ cm}^2$$

The surface area of the box of chocolates is 1540 cm^2 .

5. $A = 2bs + b^2$

$$= 2(1.0 \text{ m})(1.5 \text{ m}) + (1.0 \text{ m})^2$$

$$= 3.0 \text{ m}^2 + 1.0 \text{ m}^2$$

$$= 4.0 \text{ m}^2$$

The surface area of the pyramid is 4.0 m^2 .

6. $A = 2bs + b^2$

$$= 2(15 \text{ m})(26 \text{ m}) + (15 \text{ m})^2$$

$$= 1005 \text{ m}^2$$

The surface area of the pyramid is 1005 m^2 .

7. $A = \pi r^2 + \pi rs$

$$= (\pi)(20.0 \text{ m})^2 + (\pi)(20.0 \text{ m})(46.0 \text{ m})$$

$$\doteq 1257 \text{ m}^2 + 2890 \text{ m}^2$$

$$\doteq 4147 \text{ m}^2$$

The surface area of the sculpture is 4147 m^2 .

8. $A = \pi r^2 + \pi rs$

$$= (\pi)(24.0 \text{ cm})^2 + (\pi)(24.0 \text{ cm})(35.0 \text{ cm})$$

$$\doteq 4448 \text{ cm}^2$$

The cone has a surface area of 4448 cm^2 .

9. $A = 2\pi rh + 2\pi r^2$

$$= 2(\pi)(10.2 \text{ m})(11.5 \text{ m}) + 2(\pi)(10.2 \text{ m})^2$$

$$\doteq 737.02 \text{ m}^2 + 653.70 \text{ m}^2$$

$$\doteq 1391 \text{ m}^2$$

The surface area of the cylinder is 1391 m^2 .

10. $A = 2\pi rh$

$$= 2(\pi)(41 \text{ cm})(68 \text{ cm})$$

$$\doteq 17\,518 \text{ cm}^2$$

The area of the label is $17\,518 \text{ cm}^2$.

11. $A = 4\pi r^2$

$$= 4(\pi)(4.78 \text{ m})^2$$

$$\doteq 287 \text{ m}^2$$

The surface area of the sphere is 287 m^2 .

12. $A = 4\pi r^2$

$$= 4(\pi)(8.08 \text{ cm})^2$$

$$\doteq 820 \text{ cm}^2$$

The surface area of the sphere is 820 cm^2 .

13. $A = 2\pi rh + \pi r^2 + \frac{1}{2}(4\pi r^2)$

$$= 2\pi rh + \pi r^2 + 2\pi r^2$$

$$= 2(\pi)(23 \text{ cm})(31 \text{ cm}) + (\pi)(23 \text{ cm})^2$$

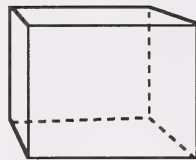
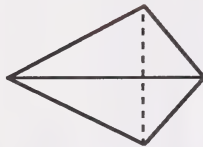
$$+ 2(\pi)(23 \text{ cm})^2$$

$$= 4480 \text{ cm}^2 + 1662 \text{ cm}^2 + 3324 \text{ cm}^2$$

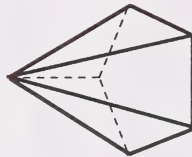
$$\doteq 9466 \text{ cm}^2$$

The surface area of the figure is 9466 cm^2 .

14. Your drawing should look like this.

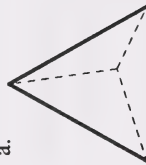


- 15.

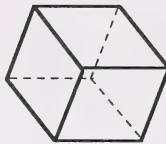


16. Your polyhedrons should look like the following.

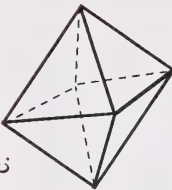
a.



b.

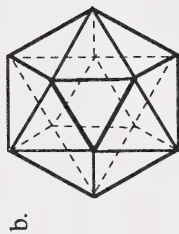
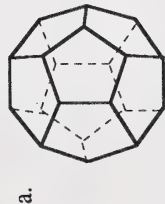


c.



17. Your polyhedrons should look like the following.

This is a challenging exercise. Do not be concerned if you are not able to construct these solids.



18. For the regular tetrahedron, regular octahedron, and regular icosahedron, the faces are equilateral triangles.

$$\begin{aligned}
 19. \quad A_f &= s^2 \\
 &= (12.0 \text{ cm})^2 \\
 &= 144 \text{ cm}^2 \\
 A &= A_f n \\
 &= (144 \text{ cm}^2)(6) \\
 &= 864 \text{ cm}^2
 \end{aligned}$$

The surface area of the regular hexahedron is 864 cm^2 .

$$\begin{aligned}
 20. \quad A_f &= \frac{\sqrt{3}}{4} s^2 \\
 &= \frac{\sqrt{3}}{4} (15 \text{ cm})^2 \\
 &\approx 97.4 \text{ cm}^2 \\
 A &= A_f n \\
 &= (97.4 \text{ cm}^2)(4) \\
 &= 390 \text{ cm}^2
 \end{aligned}$$

The surface area of the regular tetrahedron is 390 cm^2 .

$$\begin{aligned}
 21. \quad A &= A_f n \\
 &= (5.8 \text{ m}^2)12 \\
 &\approx 70 \text{ m}^2
 \end{aligned}$$

The surface area of the dodecahedron is 70 m^2 .

$$\begin{aligned}
 22. \quad A_f &= \frac{\sqrt{3}}{4} s^2 \\
 &= \frac{\sqrt{3}}{4} (7.0 \text{ cm})^2 \\
 &\approx 21.2 \text{ cm}^2 \\
 A &= A_f n \\
 &= (21.2 \text{ cm}^2)8 \\
 &\approx 170 \text{ cm}^2
 \end{aligned}$$

The surface area of the regular octahedron is 170 cm^2 .

$$23. \quad A = A_f n$$

$$= (4.2 \text{ m}^2) 20$$

$$= 84 \text{ m}^2$$

The surface area of the regular icosahedron is 84 m^2 .

$$24. \quad A = A_f n$$

$$A_f = \frac{A}{n}$$

$$= \frac{640 \text{ cm}^2}{8}$$

$$= 80 \text{ cm}^2$$

$$A_f = \frac{\sqrt{3}}{4} s^2$$

$$s^2 = \frac{4}{\sqrt{3}} A_f$$

$$= \frac{4}{\sqrt{3}} (80 \text{ cm}^2)$$

$$\doteq 184.75 \text{ cm}^2$$

$$s \doteq \sqrt{184.75 \text{ cm}^2}$$

$$\doteq 13.6 \text{ cm}$$

The length of each edge of the regular octahedron is 13.6 cm .

Extra Help

$$1. \quad A = lw$$

$$= (45 \text{ cm})(12 \text{ cm})$$

$$= 540 \text{ cm}^2$$

The area of the rectangle is 540 cm^2 .

$$2. \quad A = 4\pi r^2$$

$$= 4(\pi)(7.23 \text{ cm})^2$$

$$\doteq 657 \text{ cm}^2$$

The surface area of the sphere is 657 cm^2 .

$$3. \quad A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(12.0 \text{ cm})(26.0 \text{ cm} + 48.0 \text{ cm})$$

$$= (6.00 \text{ cm})(74.0 \text{ cm})$$

$$= 444 \text{ cm}^2$$

The area of the parallelogram is 444 cm^2 .

$$4. \quad A = \pi r^2 + \pi rs$$

$$= (\pi)(5.20 \text{ cm})^2 + (\pi)(5.20 \text{ cm})(6.80 \text{ cm})$$

$$\doteq 84.9 \text{ cm}^2 + 111.1 \text{ cm}^2$$

$$\doteq 196 \text{ cm}^2$$

The surface area of the cone is 196 cm^2 .

$$5. A = \pi r^2$$

$$= (\pi)(6.00 \text{ cm})^2$$

$$\doteq 113 \text{ cm}^2$$

The area of the circle is 113 cm^2 .

$$6. A = 2bs + b^2$$

$$= 2(12.0 \text{ cm})(24.0 \text{ cm}) + (12.0 \text{ cm})^2$$

$$= 576 \text{ cm}^2 + 144 \text{ cm}^2$$

$$= 720 \text{ cm}^2$$

The surface area of the pyramid is 720 cm^2 .

$$7. A = A_f n$$

$$= (12.5 \text{ cm}^2) 4$$

$$= 50.0 \text{ cm}^2$$

The surface area of the tetrahedron is 50.0 cm^2 .

Extensions

$$1. A = \pi dh$$

$$= (\pi)(12\ 700 \text{ km})(5066 \text{ km})$$

$$\doteq 202\ 124\ 417 \text{ km}^2$$

The area of the Torrid Zone is $202\ 124\ 417 \text{ km}^2$.

$$2. A = \pi dh$$

$$= (\pi)(12\ 700 \text{ km})(3293 \text{ km})$$

$$\doteq 131\ 384\ 861 \text{ km}^2$$

The area of each of the Temperate Zones is $131\ 384\ 861 \text{ km}^2$.

$$3. A = \pi dh$$

$$= (\pi)(12\ 700 \text{ km})(524 \text{ km})$$

$$\doteq 20\ 906\ 671 \text{ km}^2$$

The area of each of the Frigid Zones is $20\ 906\ 671 \text{ km}^2$.

$$4. r = \frac{12\ 700 \text{ km}}{2}$$

$$= 6350 \text{ km}$$

$$A = 4\pi r^2$$

$$= 4(\pi)(6350 \text{ km})^2$$

$$\doteq 506\ 707\ 479 \text{ km}^2$$

The earth's surface area is $506\ 707\ 479 \text{ km}^2$.

$$\frac{2(131\ 384\ 861 \text{ km}^2)}{506\ 707\ 479 \text{ km}^2} \times 100\% \doteq 52\%$$

The Temperate Zones are 52% of the earth's surface.



Exploring Topic 3

Activity 1

Calculate the volume of prisms and pyramids.

1. $l = 5 \text{ cm}$

$w = 4 \text{ cm}$

$h = 4 \text{ cm}$

$V = lwh$

$= (5 \text{ cm})(4 \text{ cm})(4 \text{ cm})$

$= 80 \text{ cm}^3$

The volume of the solid is 80 cm^3 .

2. $V = lwh$

$= (19.4 \text{ cm})(5.30 \text{ cm})(29.9 \text{ cm})$

$\approx 3074 \text{ cm}^3$

The volume of the box is 3074 cm^3 .

3. $V = lwh$

$= (4.5 \text{ m})(3.6 \text{ m})(3.0 \text{ m})$

$= 48.6 \text{ m}^3$

The volume of the room is less than 50 m^3 .

The humidifier is fine for the room.

4. $V_1 = lwh$

$= (20.0 \text{ cm})(15.0 \text{ cm})(10.0 \text{ cm})$

$= 3000 \text{ cm}^3$

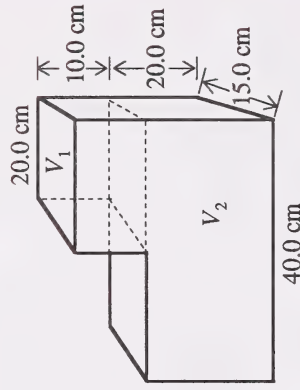
$V_2 = (40.0 \text{ cm})(15.0 \text{ cm})(20.0 \text{ cm})$

$= 12\,000 \text{ cm}^3$

$V = V_1 + V_2$

$= 3000 \text{ cm}^3 + 12\,000 \text{ cm}^3$

$= 15\,000 \text{ cm}^3$



The volume of the solid is $15\,000 \text{ cm}^3$.

5. $V = lwh$

$= (100 \text{ cm})(55 \text{ cm})(50 \text{ cm})$

$= 275\,000 \text{ cm}^3$

$= 275\,000 \text{ cm}^3 \times \frac{1 \text{ L}}{1000 \text{ cm}^3}$

$= 275 \text{ L}$

To fill the aquarium, 275 L of water are needed.

$$\begin{aligned}
 6. \quad V &= (100 \text{ cm})(100 \text{ cm})(100 \text{ cm}) \\
 &= 1\,000\,000 \text{ cm}^3 \\
 &= 1\,000\,000 \text{ cm}^3 \times \frac{1 \text{ L}}{1000 \text{ cm}^3} = 1000 \text{ L}
 \end{aligned}$$

There are 1000 L in 1 m^3 .

$$7. \quad B = \frac{1}{2}bh, \text{ where } h \text{ is the height of the triangular base.}$$

$$= \frac{1}{2}(5.9 \text{ cm})(6.2 \text{ cm})$$

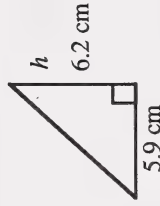
$$= 18.29 \text{ cm}^2$$

$V = Bh$, where h is the height of the prism.

$$= (18.29 \text{ cm}^2)(6.8 \text{ cm})$$

$$\doteq 124 \text{ cm}^3$$

The volume of the prism is 124 cm^3 .



$$8. \quad B = \frac{1}{2}h(a+b), \text{ where } h \text{ is the height of the trapezoidal base.}$$

$$= \frac{1}{2}(2.75 \text{ cm})(4.24 \text{ cm} + 8.60 \text{ cm})$$

$$= 17.655 \text{ cm}^2$$

$V = Bh$, where h is the height of the prism.

$$= (17.655 \text{ cm}^2)(3.05 \text{ cm})$$

$$\doteq 53.8 \text{ cm}^3$$

The volume of the prism is 53.8 cm^3 .

$$9. \quad B = \frac{1}{2}bh, \text{ where } h \text{ is the height of the triangular base.}$$

$$= \frac{1}{2}(18 \text{ m})(6 \text{ m})$$

$$= 54 \text{ m}^2$$

$V = Bh$, where h is the height of the prism-shaped space.

$$= (54 \text{ m}^2)(21 \text{ m})$$

$$= 1134 \text{ m}^3$$

The volume of the attic is 1134 m^3 .

$$10. \quad B = \frac{\sqrt{3}}{4}s^2$$

$$= \frac{\sqrt{3}}{4}(3.75 \text{ m})^2$$

$$\doteq 6.089 \text{ m}^2$$

$V = Bh$

$$= (6.089 \text{ m}^2)(1.25 \text{ m})$$

$$\doteq 7.61 \text{ m}^3$$

The volume of the prism is 7.61 m^3 .

11. You should find that the prism will hold three pyramids full of rice. Remember to tap the prism to settle the rice; otherwise, the rice will be rounded.

$$12. \quad V = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(12.4 \text{ cm})^2(9.8 \text{ cm})$$

$$\approx 502 \text{ cm}^3$$

The volume of the pyramid is 502 cm^3 .

$$13. \quad V = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(12.0 \text{ m})^2(9.0 \text{ m})$$

$$= 432 \text{ m}^3$$

The volume of the pyramid is 432 m^3 .

$$14. \quad V = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(240 \text{ m})^2(148 \text{ m})$$

$$= 2\,841\,600 \text{ m}^3$$

The volume of the pyramid is $2\,841\,600 \text{ m}^3$.

$$15. \quad V = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(25.7 \text{ m})^2(24.1 \text{ m})$$

$$\approx 5306 \text{ m}^3$$

The volume of the temperate pyramid is 5306 m^3 .

$$16. \quad V_1 = bwh$$

$$= (5.5 \text{ m})(5.5 \text{ m})(6.0 \text{ m})$$

$$= 181.5 \text{ m}^3$$

$$V_2 = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(5.5 \text{ m})^2(3.1 \text{ m})$$

$$\approx 31.3 \text{ m}^3$$

$$V = V_1 + V_2$$

$$= 181.5 \text{ m}^3 + 31.3 \text{ m}^3$$

$$= 212.8 \text{ m}^3$$

The volume of the container is 213 m^3 .

$$17. \quad V = \frac{1}{3}b^2h$$

$$= \frac{1}{3}(1.2 \text{ m})^2(2.0 \text{ m})$$

$$= 0.96 \text{ m}^3$$

The volume of the hopper is 0.96 m^3 .

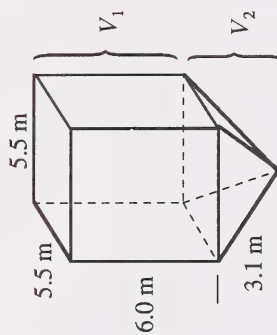
$$18. \quad V = s^3$$

$$s = \sqrt[3]{V}$$

$$= \sqrt[3]{25.0 \text{ m}^3}$$

$$\approx 2.92 \text{ m}$$

The length of each cube side is 2.92 m .



Activity 2

Calculate the volume of cylinders and cones.

1. $V = \pi r^2 h$

$$= (\pi)(14 \text{ cm})^2(20 \text{ cm})$$

$$\approx 12\,315 \text{ cm}^3$$

The volume of the cylinder is $12\,315 \text{ cm}^3$.

2. $V = \pi r^2 h$

$$= (\pi)(45.0 \text{ cm})^2(95.0 \text{ cm})$$

$$\approx 604\,363 \text{ cm}^3$$

The cylinder volume is $604\,363 \text{ cm}^3$.

3. $V = 10 \text{ L}$

$$= 10\,000 \text{ cm}^3$$

$$r = \frac{11.3 \text{ cm}}{2}$$

$$= 5.65 \text{ cm}$$

$$V = \pi r^2 h$$

$$h = \frac{V}{\pi r^2}$$

$$= \frac{10\,000 \text{ cm}^3}{(\pi)(5.65 \text{ cm})^2}$$

$$\approx 100 \text{ cm}$$

The liquid would rise to 100 cm.

4. $r = \frac{4.50 \text{ cm}}{2}$

$$= 2.25 \text{ cm}$$

$$V = \pi r^2 h$$

$$= (\pi)(2.25 \text{ cm})^2(15.0 \text{ cm})$$

$$\approx 239 \text{ cm}^3$$

The engine displacement is 239 cm^3 .

5. $V = \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} (\pi)(2.50 \text{ m})^2(3.00 \text{ m})$$

$$\approx 19.6 \text{ m}^3$$

The volume of the cone is 19.6 m^3 .

6. $r = 3.00 \text{ cm}$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} (\pi) (3.00 \text{ cm})^2 (15.0 \text{ cm})$$

$$\doteq 141 \text{ cm}^3$$

The volume is 141 cm^3 .

7. $r = 4.00 \text{ cm}$

$$h^2 + (4.00 \text{ cm})^2 = 5.00 \text{ cm}^2$$

$$h^2 = (5.00 \text{ cm})^2 - (4.00 \text{ cm})^2$$

$$= 25.00 \text{ cm}^2 - 16.00 \text{ cm}^2$$

$$= 9.00 \text{ cm}^2$$

$$h = \sqrt{9.00 \text{ cm}^2}$$

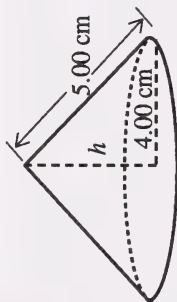
$$= 3.00 \text{ cm}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} (\pi) (4.00 \text{ cm})^2 (3.00 \text{ cm})$$

$$\doteq 50.3 \text{ cm}^3$$

The volume of the cone is 50.3 cm^3 .



8. For the cylindrical part,

$$V_1 = \pi r^2 h$$

$$= (\pi) (60.0 \text{ cm})^2 (50.0 \text{ cm})$$

$$= 565\,487 \text{ cm}^3$$

For the cone part,

$$h = 230 \text{ cm} - 50 \text{ cm}$$

$$= 180 \text{ cm}$$

$$V_2 = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} (\pi) (60.0 \text{ cm})^2 (180 \text{ cm})$$

$$= 678\,584 \text{ cm}^3$$

$$V = 565\,487 \text{ cm}^3 + 678\,584 \text{ cm}^3$$

$$= 1\,244\,071 \text{ cm}^3 \text{ or } 1.24 \text{ m}^3$$

The hopper volume is $1\,244\,071 \text{ cm}^3$ or 1.24 m^3 .

9. For the cone,

$$\begin{aligned} V_2 &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} (\pi) (0.8 \text{ cm})^2 (2.8 \text{ cm}) \\ &\doteq 1.88 \text{ cm}^3 \end{aligned}$$

For the removed part,

$$\begin{aligned} V_1 &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} (\pi) (0.2 \text{ cm})^2 (0.7 \text{ cm}) \\ &\doteq 0.03 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V &= V_2 - V_1 \\ &= 1.88 \text{ cm}^3 - 0.03 \text{ cm}^3 \\ &= 1.85 \text{ cm}^3 \end{aligned}$$

The figure has a volume of 1.85 cm^3 .

$$\begin{aligned} 10. \quad V &= \pi r^2 h \\ &= (\pi) \left(\frac{6.00 \text{ m}}{2} \right)^2 (35.0 \text{ m}) \\ &\doteq 990 \text{ m}^3 \end{aligned}$$

The silo can store 990 m^3 of silage.

Activity 3

Solve problems involving the volume of spheres.

$$\begin{aligned} 1. \quad V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} (\pi) (0.75 \text{ m})^3 \\ &\doteq 1.77 \text{ m}^3 \end{aligned}$$

The volume of the sphere is 1.77 m^3 .

$$\begin{aligned} 2. \quad V &= \frac{4}{3} \pi r^3 \\ r &= \frac{1.15 \text{ m}}{2} \\ &= 0.575 \text{ m} \\ &= \frac{4}{3} (\pi) (0.575 \text{ m})^3 \\ &\doteq 0.796 \text{ m}^3 \end{aligned}$$

The volume of the ball is 0.796 m^3 .

$$3. \quad r = \frac{6.50 \text{ cm}}{2}$$

$$= 3.25 \text{ cm}$$

For each ball,

$$V_1 = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} (\pi) (3.25 \text{ cm})^3$$

$$\approx 144 \text{ cm}^3$$

For the cylinder,

$$h = 3d$$

$$= 3(6.50 \text{ cm})$$

$$= 19.50 \text{ cm}$$

$$V_2 = \pi r^2 h$$

$$= (\pi) (3.25 \text{ cm})^2 (19.50 \text{ cm})$$

$$\approx 647 \text{ cm}^3$$

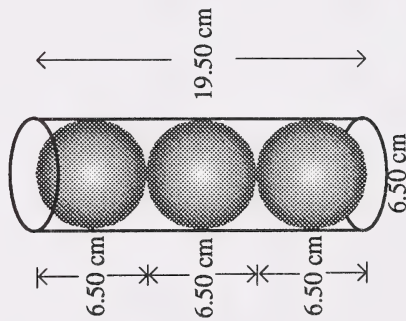
For the air around the balls,

$$V = V_2 - 3(V_1)$$

$$= 647 \text{ cm}^3 - 3(144 \text{ cm}^3)$$

$$= 215 \text{ cm}^3$$

There are 215 cm^3 of air in the container and outside the balls.



$$4. \quad V = \frac{4}{3} \pi r^3$$

$$r^3 = \frac{3V}{4\pi}$$

$$= \left(\frac{3}{4} \right) \frac{113 \text{ cm}^3}{\pi}$$

$$\approx 26.98 \text{ cm}^3$$

$$r \approx \sqrt[3]{26.98 \text{ cm}^3}$$

$$\approx 3 \text{ cm}$$

The radius of the sphere is 3 cm.

5. For a sphere of radius 1 m,

$$V_1 = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} (\pi) (1 \text{ m})^3$$

$$\approx 4.19 \text{ m}^3$$

For a sphere of radius 2 m,

$$V_2 = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} (\pi) (2 \text{ m})^3$$

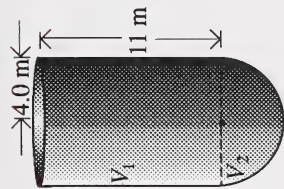
$$\approx 33.5 \text{ m}^3$$

$$\frac{V_2}{V_1} = \frac{33.5 \text{ m}^3}{4.19 \text{ m}^3}$$

$$= 8$$

Doubling the radius of a sphere increases the volume by a factor of eight.

$$\begin{aligned}
 6. \quad V_1 &= \pi r^2 h \\
 &= (\pi)(4.0)^2(11 \text{ m}) \\
 &\approx 553 \text{ m}^3 \\
 V_2 &= \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \\
 &= \frac{2}{3} (\pi)(4.0 \text{ m})^3 \\
 &\approx 134 \text{ m}^3
 \end{aligned}$$



$$\begin{aligned}
 V &= V_1 + V_2 \\
 &= 553 \text{ m}^3 + 134 \text{ m}^3 \\
 &= 687 \text{ m}^3
 \end{aligned}$$

The volume of the container is 687 m^3 .

7.

	Rectangular Prism	Sphere	Cylinder
a. Surface area	$ \begin{aligned} A &= 2[(8.00 \text{ cm})(8.73 \text{ cm}) \\ &\quad + (8.00 \text{ cm})(3.00 \text{ cm}) \\ &\quad + (8.73 \text{ cm})(3.00 \text{ cm})] \\ &\approx 240 \text{ cm}^2 \end{aligned} $	$ \begin{aligned} A &= 4\pi r^2 \\ &= 4(\pi)(4.37 \text{ cm})^2 \\ &\approx 240 \text{ cm}^2 \end{aligned} $	$ \begin{aligned} A &= 2\pi r h + 2\pi r^2 \\ &= 2(\pi)(5.00 \text{ cm})(2.64 \text{ cm}) \\ &\quad + 2(\pi)(5.00 \text{ cm})^2 \\ &\approx 240 \text{ cm}^2 \end{aligned} $
b. Volume	$ \begin{aligned} V &= lwh \\ &= (8.73 \text{ cm})(3.00 \text{ cm})(8.00 \text{ cm}) \\ &\approx 210 \text{ cm}^3 \end{aligned} $	$ \begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}(\pi)(4.37 \text{ cm})^3 \\ &\approx 350 \text{ cm}^3 \end{aligned} $	$ \begin{aligned} V &= \pi r^2 h \\ &= (\pi)(5.00 \text{ cm})^2(2.64 \text{ cm}) \\ &\approx 207 \text{ cm}^3 \end{aligned} $

c. The sphere has the greatest volume.

8.

	Cube	Cone	Sphere
a. Surface area	$A = A_f n$ $= (5.32 \text{ cm})^2 6$ $\approx 170 \text{ cm}^2$	$A = \pi r^2 + \pi r s$ $= (\pi)(4.50 \text{ cm})^2 + (\pi)(4.50 \text{ cm})(7.50 \text{ cm})$ $\approx 63.6 \text{ cm}^2 + 106.0 \text{ cm}^2$ $\approx 170 \text{ cm}^2$	$A = 4\pi r^2$ $= 4(\pi)(3.67 \text{ cm})^2$ $\approx 169 \text{ cm}^2$
b. Volume	$V = s^3$ $= (5.32 \text{ cm})^3$ $\approx 151 \text{ cm}^3$	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}(\pi)(4.50 \text{ cm})^2(6.00 \text{ cm})$ $\approx 127 \text{ cm}^3$	$V = \frac{4}{3}\pi r^3$ $= \frac{4}{3}(\pi)(3.67 \text{ cm})^3$ $\approx 207 \text{ cm}^3$

c. The sphere has the greatest volume.

9. The sphere has the greatest volume for a given surface area.

Extra Help

$$\begin{aligned}1. \quad B &= \frac{1}{2}h(a+b) \\&= \frac{1}{2}(12.0 \text{ m})(19.0 \text{ m} + 42.0 \text{ m}) \\&= 366 \text{ m}^2\end{aligned}$$

$$\begin{aligned}V &= Bh \\&= (366 \text{ m}^2)(18 \text{ m}) \\&= 6588 \text{ m}^3\end{aligned}$$

The volume of the prism is 6588 m^3 .

$$\begin{aligned}2. \quad V &= \frac{1}{3}b^2h \\&= \frac{1}{3}(15.0 \text{ cm})^2(9.00 \text{ cm}) \\&= 675 \text{ cm}^3\end{aligned}$$

The volume of the prism is 675 cm^3 .

$$\begin{aligned}3. \quad V &= \pi r^2h \\&= (\pi)(0.500 \text{ m})^2(1.00 \text{ m}) \\&= 0.785 \text{ m}^3\end{aligned}$$

The volume of the cylinder is 0.785 m^3 .

$$\begin{aligned}4. \quad V &= \frac{1}{3}(\pi r^2)h \\&= \frac{1}{3}(75.0 \text{ cm}^2)(12.0 \text{ cm}) \\&= 300 \text{ cm}^3\end{aligned}$$

The volume of the cone is 300 cm^3 .

$$\begin{aligned}5. \quad V &= \frac{4}{3}\pi r^3 \\&= \frac{4}{3}(\pi)(6.00 \text{ m})^3 \\&= 905 \text{ m}^3\end{aligned}$$

The volume of the sphere is 905 m^3 .

Extensions

$$\begin{aligned}1. \quad V &= \frac{1}{3}\pi h(a^2 + ab + b^2) \\&= \frac{1}{3}(\pi)(1.50 \text{ cm}) \\&\quad \left[(1.00 \text{ cm})^2 + (1.00 \text{ m})(1.10 \text{ cm}) + (1.10 \text{ cm})^2 \right] \\&= (\pi)(0.50 \text{ cm})(3.31 \text{ cm}^2) \\&= 5.20 \text{ cm}^3\end{aligned}$$

The volume of the frustum is 5.20 cm^3 .

$$\begin{aligned}
 2. \quad V &= \frac{1}{3} \pi h (0^2 + 0b + b^2) \\
 &= \frac{1}{3} \pi h (b^2) \\
 &= \frac{1}{3} \pi b^2 h
 \end{aligned}$$

This formula applies to a cone with a radius equal to b .

$$\begin{aligned}
 3. \quad V &= \frac{1}{3} \pi h (a^2 + aa + a^2) \\
 &= \frac{1}{3} \pi h (3a^2) \\
 &= \pi a^2 h
 \end{aligned}$$

This formula applies to a cylinder with a radius equal to a .



Exploring Topic 4

Activity 1

Determine and use the relationship between the sum of the angles and the number of sides in a polygon.

1.

Figure	Number of Sides	Angle Sum ° (degrees)	Alternate Expression of Angle Sum
Triangle	3	180°	180° (3 - 2)
Quadrilateral	4	360°	180° (4 - 2)
Pentagon	5	540°	180° (5 - 2)
Hexagon	6	720°	180° (6 - 2)
Heptagon	7	900°	180° (7 - 2)

2. The angle sum increases by 180° for every additional side.

3. See the last column of the table.

4. The sum of the angles of a polygon of n sides is $180^\circ (n - 2)$.

5. a. $180^\circ (10 - 2) = 1440^\circ$

b. $180^\circ (4 - 2) = 360^\circ$

c. $180^\circ (50 - 2) = 8640^\circ$

d. $180^\circ (16 - 2) = 2520^\circ$

e. $180^\circ (20 - 2) = 3240^\circ$

6. $180^\circ(n-2) = \text{angle sum}$

$$(n-2) = \frac{\text{angle sum}}{180^\circ}$$

$$n = \frac{\text{angle sum}}{180^\circ} + 2$$

a. $n = \frac{1800^\circ}{180^\circ} + 2$
 $= 12$

b. $n = \frac{2160^\circ}{180^\circ} + 2$
 $= 14$

c. $n = \frac{360^\circ}{180^\circ} + 2$
 $= 4$

d. $n = \frac{180^\circ}{180^\circ} + 2$
 $= 3$

e. $n = \frac{1620^\circ}{180^\circ} + 2$
 $= 11$

7. $n = 4$

$$\text{angle sum} = 180^\circ(n-2)$$

$$= 180^\circ(6-2)$$

$$= 720^\circ$$

Let x be the remaining angle.

$$\text{Then, } x + 125^\circ + 145^\circ + 95^\circ + 112^\circ + 81^\circ = 720^\circ$$

$$x + 558^\circ = 720^\circ$$

$$x = 720^\circ - 558^\circ$$

$$= 162^\circ.$$

The measure of the sixth angle is 162° .

8. $n = 4$

$$\text{angle sum} = 180^\circ(n-2)$$

$$= 180^\circ(4-2)$$

$$= 360^\circ$$

Let x be the fourth angle.

$$\text{Then, } x + 85^\circ + 105^\circ + 97^\circ = 360^\circ$$

$$x + 287^\circ = 360^\circ$$

$$x = 360^\circ - 287^\circ$$

$$= 73^\circ.$$

The measure of the fourth angle is 73° .

$$\begin{aligned}
 9. \text{ angle sum} &= 180^\circ(n-2) \\
 &= 180^\circ(10-2) \\
 &= 1440^\circ
 \end{aligned}$$

$$\text{Each angle is } \frac{1440^\circ}{10} = 144^\circ.$$

The measure of each of the angles of the regular decagon is 144° .

$$\begin{aligned}
 10. \text{ angle sum} &= 180^\circ(n-2) \\
 &= 180^\circ(7-2) \\
 &= 900^\circ
 \end{aligned}$$

$$\text{Each angle is } \frac{900^\circ}{7} = 129^\circ.$$

The measure of each of the angles of the regular heptagon is 129° .

Activity 2

Determine and use the relationship between the number of sides and the number of diagonals in a polygon.

Figure	Number of Sides	Number of Diagonals	Alternate Expression of Number of Diagonals
Triangle	3	0	$\frac{(3)(0)}{2} = 0$
Quadrilateral	4	2	$\frac{(4)(1)}{2} = 2$
Pentagon	5	5	$\frac{(5)(2)}{2} = 5$
Hexagon	6	9	$\frac{(6)(3)}{2} = 9$
Heptagon	7	14	$\frac{(7)(4)}{2} = 14$
Octagon	8	20	$\frac{(8)(5)}{2} = 20$

- See the last column of the table.
- One factor equals the number of sides. The other factor is three less.
- The number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}$.

5. The number of diagonals for each is as follows.

$$\text{a. } \frac{n(n-3)}{2} = \frac{(9)(9-3)}{2} = 27$$

$$\text{b. } \frac{n(n-3)}{2} = \frac{(10)(10-3)}{2} = 35$$

$$\text{c. } \frac{n(n-3)}{2} = \frac{(14)(14-3)}{2} = 77$$

$$\text{d. } \frac{n(n-3)}{2} = \frac{(20)(20-3)}{2} = 170$$

6. The number of diagonals for each is as follows.

$$\text{a. } \frac{n(n-3)}{2} = \frac{12(12-3)}{2} = 54$$

$$\text{b. } \frac{n(n-3)}{2} = \frac{13(13-3)}{2} = 65$$

$$\text{c. } \frac{n(n-3)}{2} = \frac{15(15-3)}{2} = 90$$

$$\text{d. } \frac{n(n-3)}{2} = \frac{16(16-3)}{2} = 104$$

7. The number of diagonals is 44.

$$\frac{n(n-3)}{2} = 44$$

$$n(n-3) = 88$$

$$n^2 - 3n - 88 = 0$$

$$(n-11)(n+8) = 0$$

$$n-11 = 0$$

$$n+8 = 0$$

$$n = 11$$

$$n = -8$$

$n = -8$ does not apply.

The polygon has 11 sides.

8. The number of the paths across the lawn equals the number of diagonals of a pentagon.

$$n = 5$$

$$\text{Number of diagonals} = \frac{n(n-3)}{2} = \frac{5(5-3)}{2} = 5$$

Five straight paths can be taken across the central lawn to connect the sidewalks.

Activity 3

Determine the relationship among the numbers of edges, vertices, and faces in a polyhedron.

Answers for questions 1 through 4 are presented in this chart.

Figure	Number of Faces (F)	Number of Vertices (V)	Number of Edges (E)
Pyramid	5	5	8
Rectangular prism	6	8	12
Triangular prism	5	6	9
Regular tetrahedron	4	4	6
Regular hexahedron (cube)	6	8	12
Regular octahedron	8	6	12
Figure A	9	9	16
Figure B	8	12	18
Figure C	8	12	18

5. $F + V - E = 2$

6. $F + V - E = 2$
 $E = F + V - 2$
 $= 62 + 60 - 2$
 $= 120$

The rhombicosidodecahedron has 120 edges.

Activity 4

Determine the relationship between the number of lateral faces and the number of diagonals in a prism.

Answers for questions 1 through 3 are presented in this chart.

Prism Type	Number of Lateral Faces (n)	Number of Diagonals
Triangular prism	3	0
Rectangular prism	4	4
Pentagonal prism	5	10
Hexagonal prism	6	18
Heptagonal prism	7	28

4. The prism has twice the number of diagonals as the related polygon.

5. The number of diagonals in a prism having n lateral faces is $\frac{n(n-3)}{2}$.

Extra Help

$$1. \quad \frac{1}{2}n(n-3) = \frac{1}{2}(12)(12-3) \\ = 54$$

A dodecagon has 54 diagonals.

$$2. \quad 180^\circ(n-2) = 180^\circ(12-2) \\ = 1800^\circ$$

The angle sum of a dodecagon is 1800° .

$$3. \quad n(n-3) = 12(12-3) \\ = 108$$

The prism has 108 diagonals.

$$4. \quad F + V - E = 14 + 24 - 36 \\ = 2 \text{ as it is supposed to be.}$$

Extensions

1.

Group Size	Number of Different Handshakes
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45

In a group of n individuals, there can be $\frac{1}{2}n(n-1)$ different handshakes.

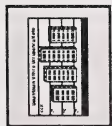
2. The number of diagonals in an n -sided polygon is $\left[\frac{1}{2}n(n-1) \right] - n$.

3. You must show that $\left[\frac{1}{2}n(n-1)\right] - n$ and $\frac{1}{2}n(n-3)$ are equivalent.

L S	R S
$\left[\frac{1}{2}n(n-1)\right] - n$	$\frac{1}{2}n(n-3)$
$= \left(\frac{1}{2}n^2 - \frac{1}{2}n\right) - n$	$= \frac{1}{2}n^2 - \frac{3}{2}n$
$= \frac{1}{2}n^2 - \frac{3}{2}n$	

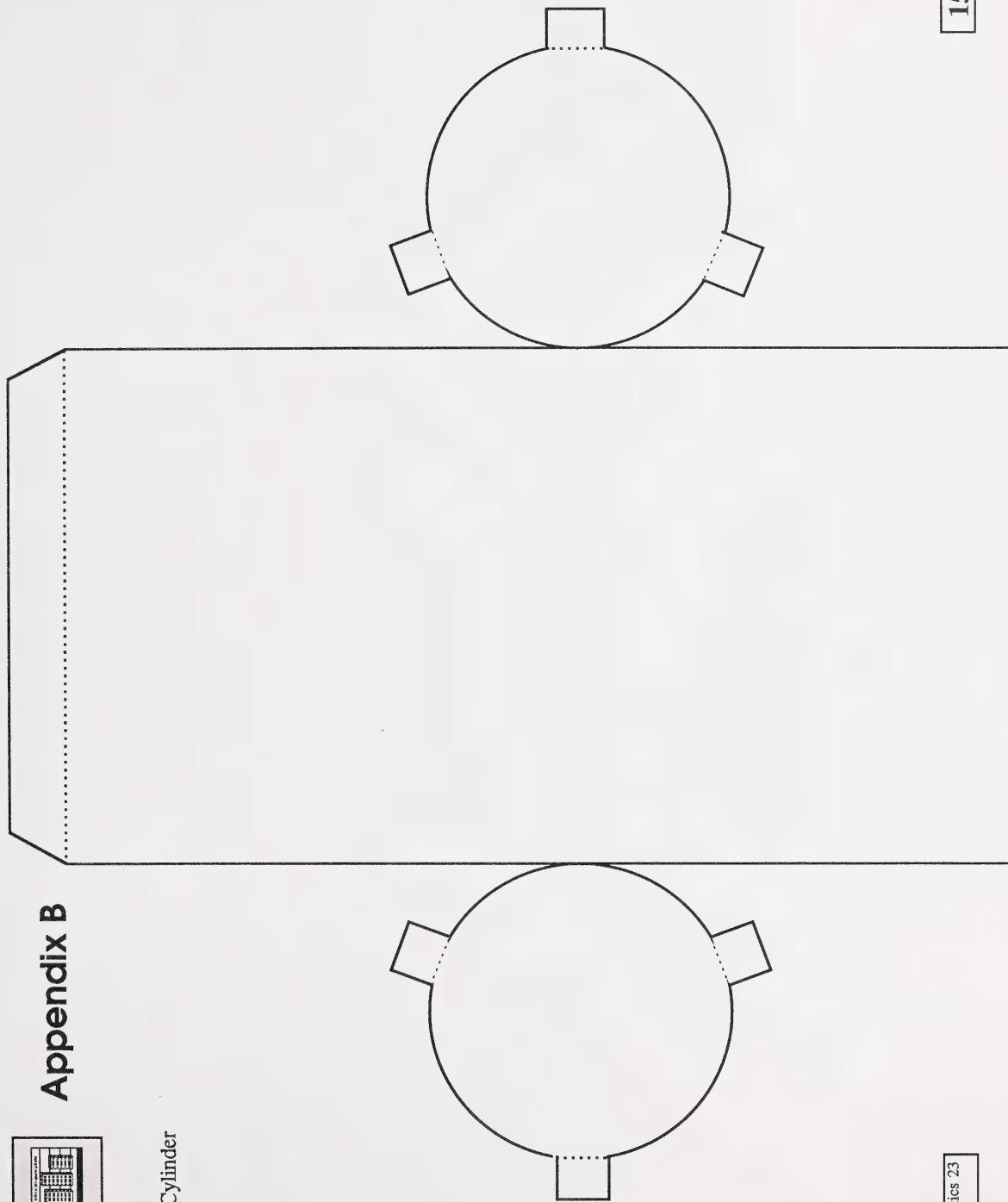
$$L S = R S$$

4. When determining the number of handshakes, you counted the adjacent persons. These would not correspond to diagonals, but rather to segments of a polygon.

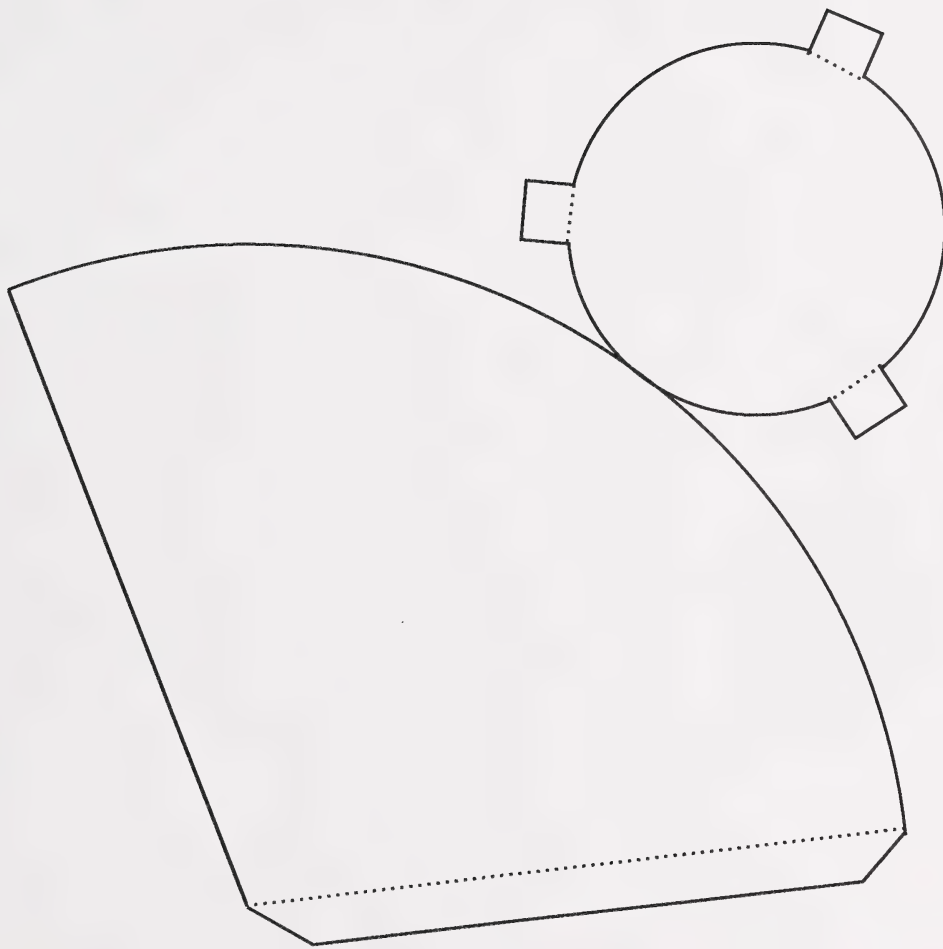


Appendix B

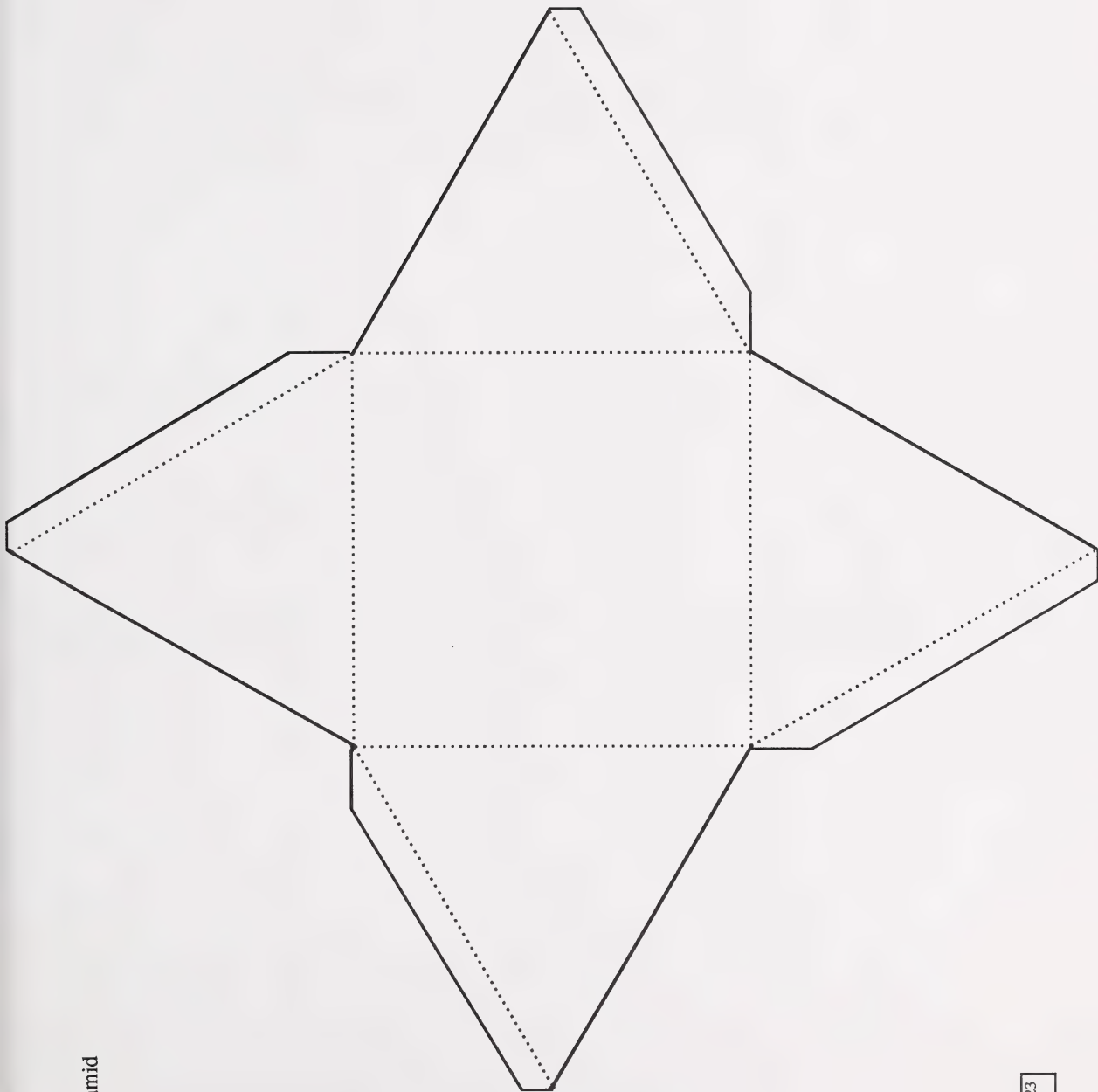
1. Cylinder



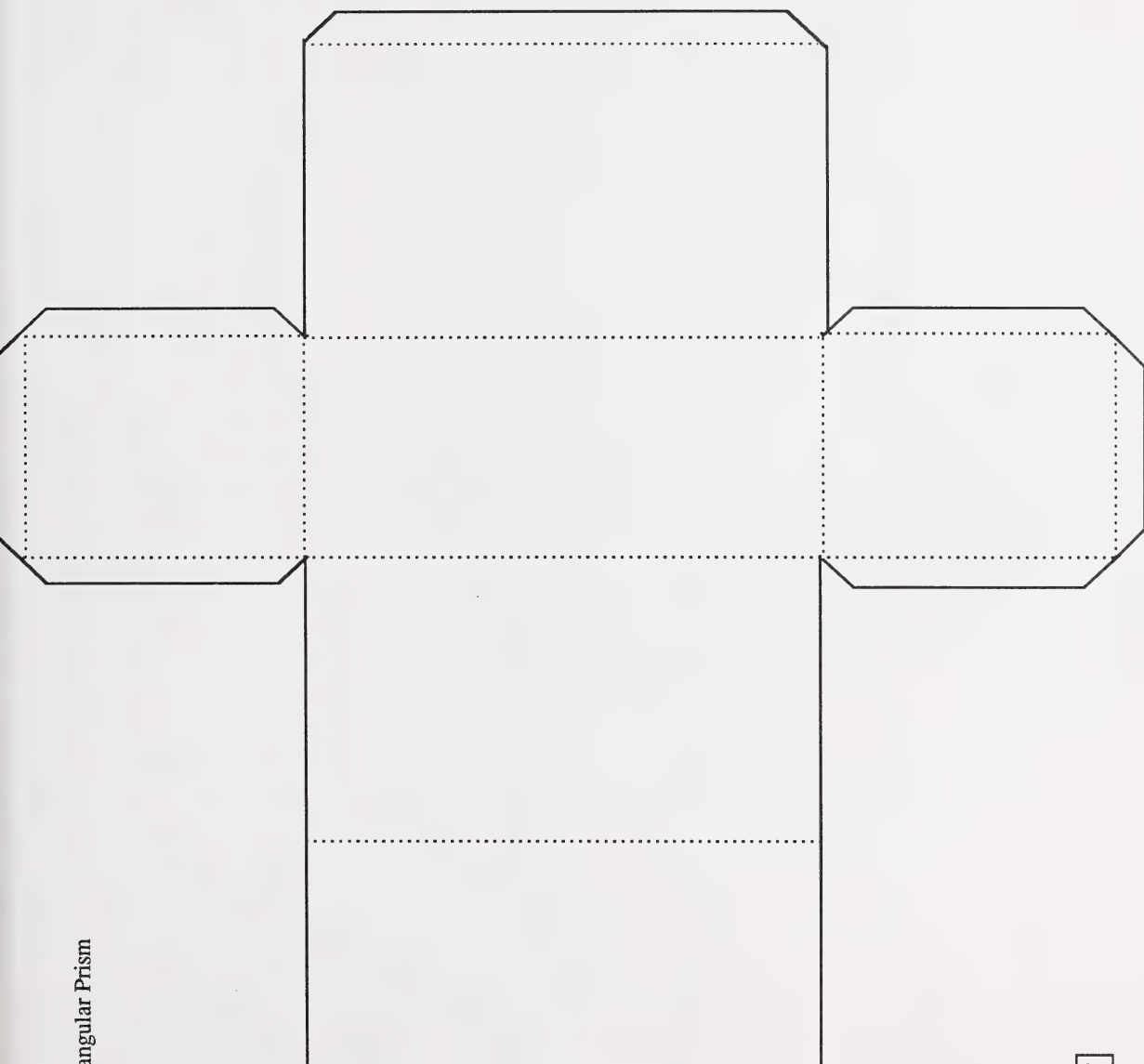
2. Cone



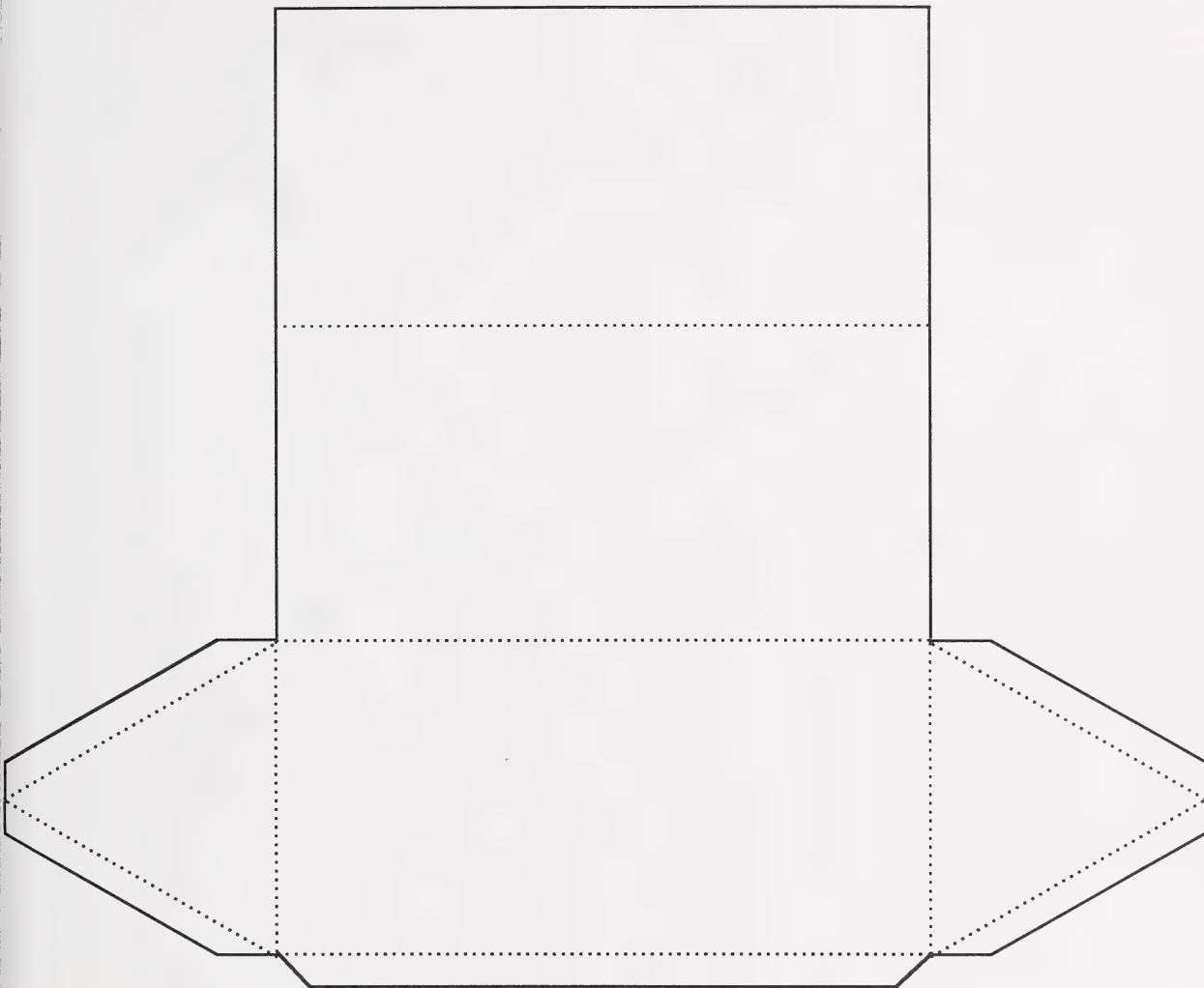
3. Pyramid



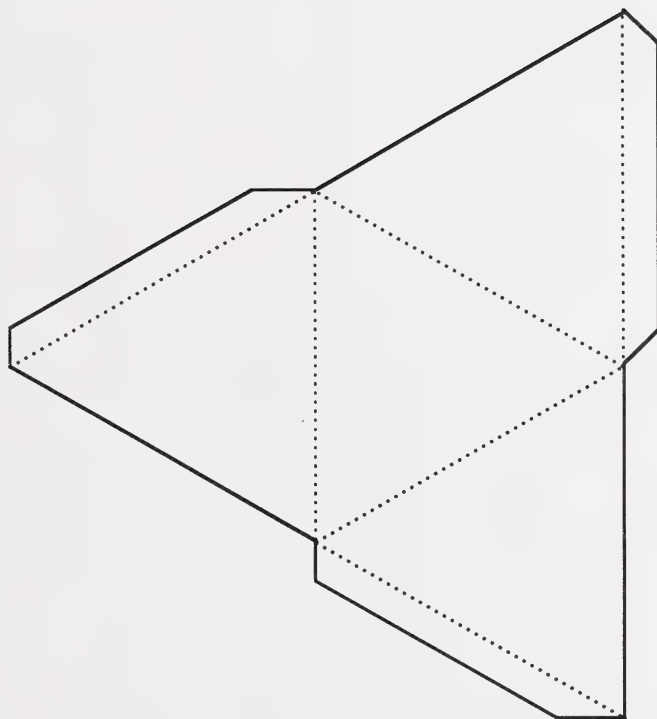
4. Rectangular Prism



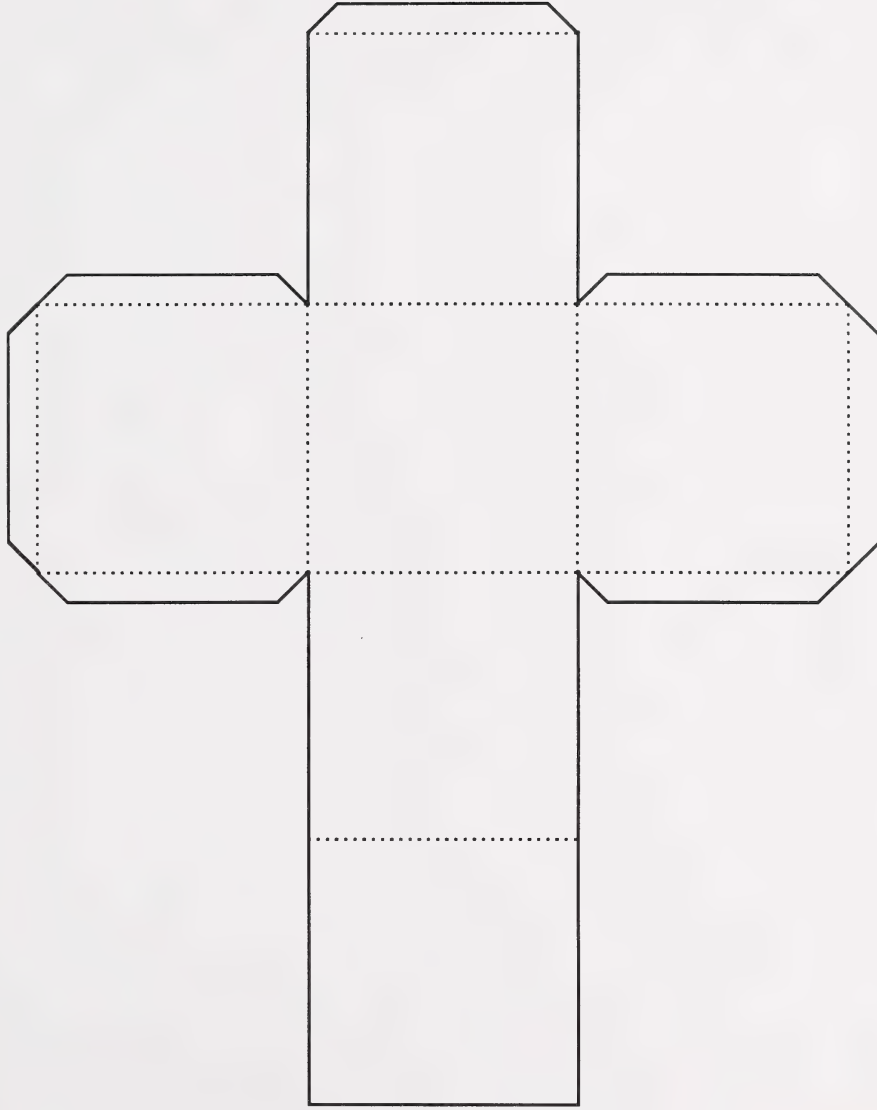
5. Triangular Prism



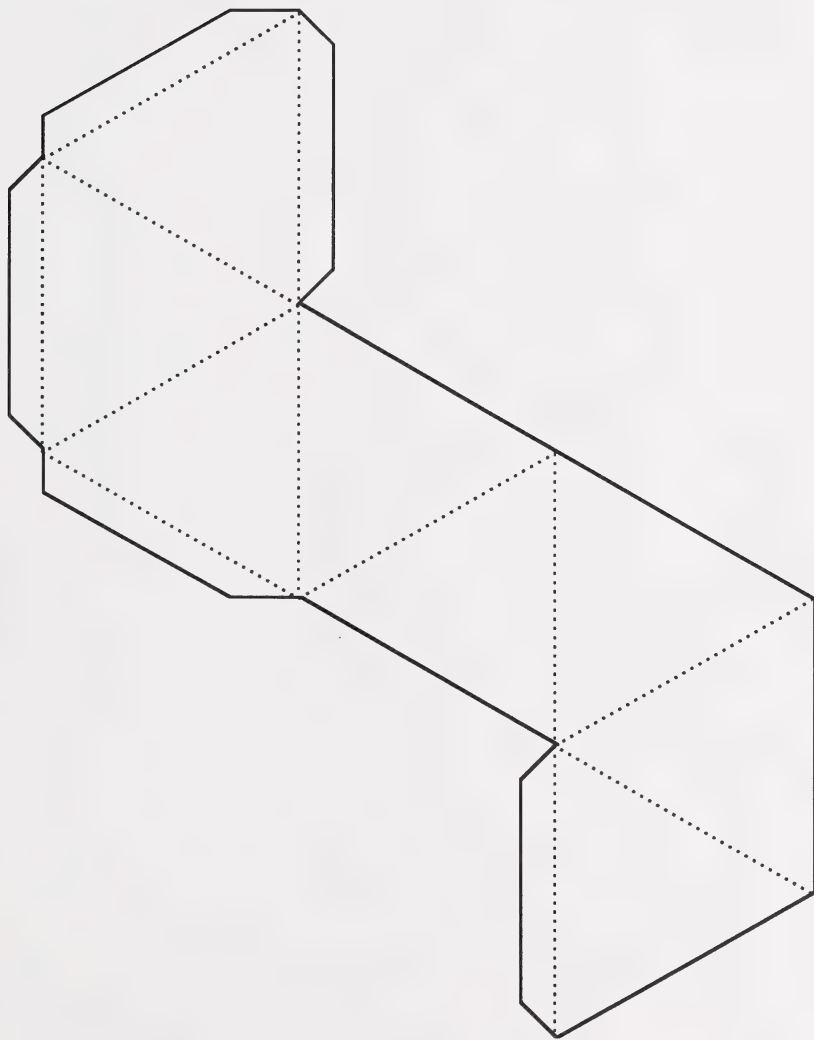
6. Regular Tetrahedron



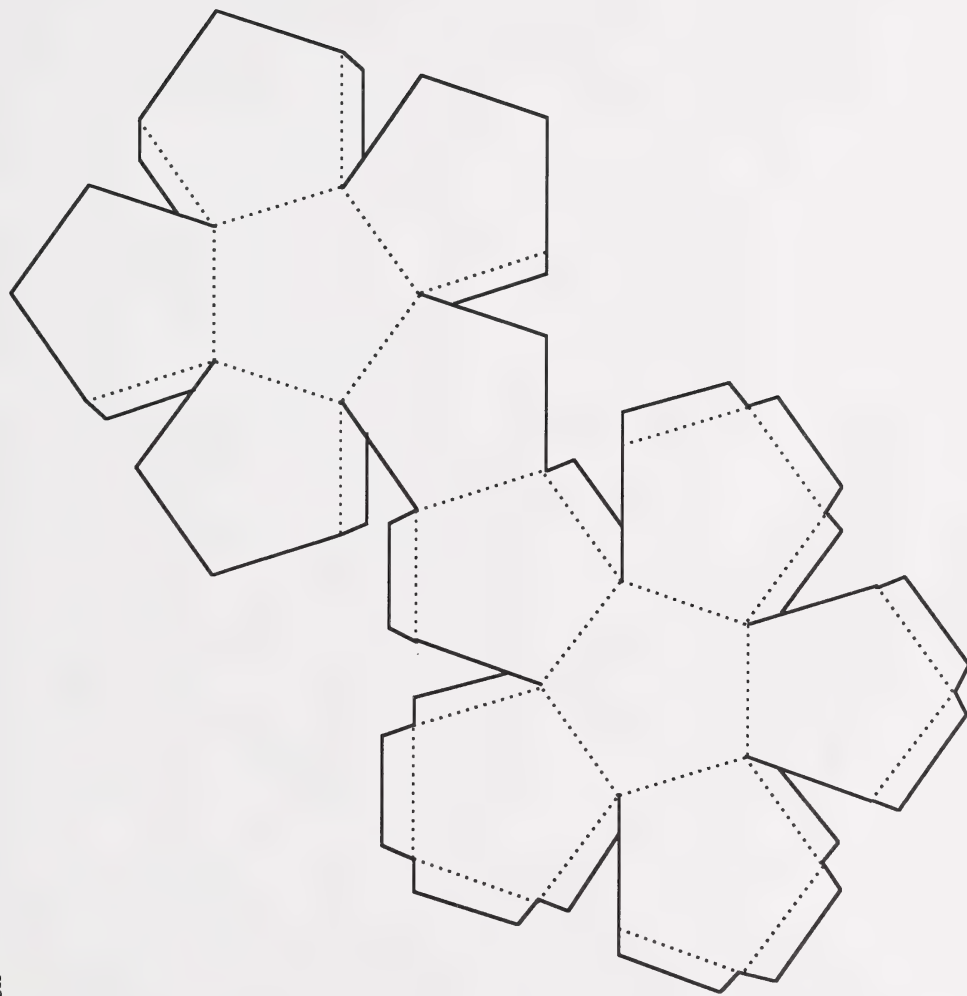
7. Regular Hexahedron



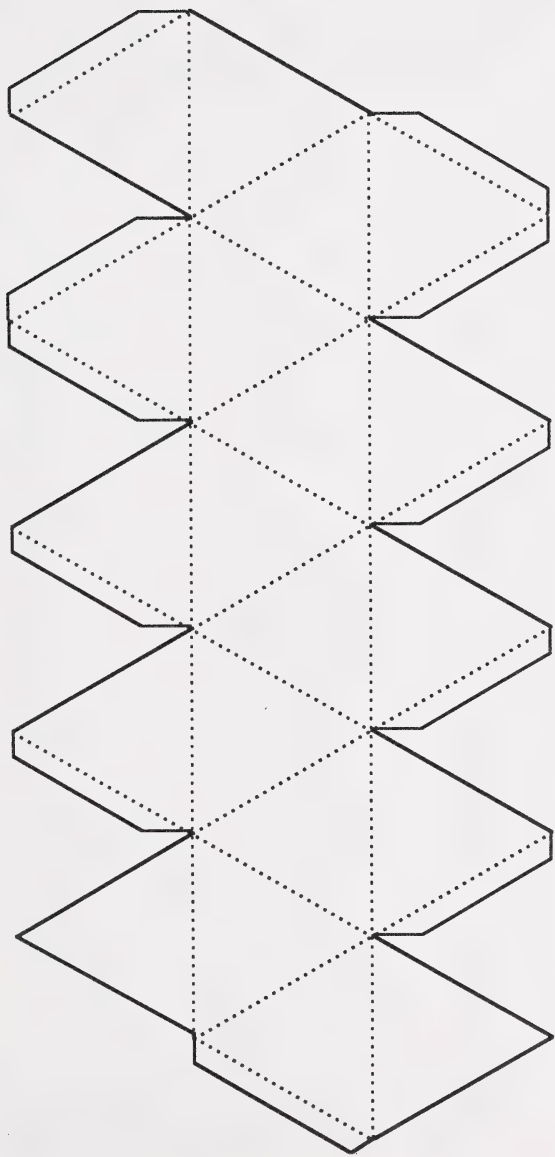
8. Regular Octahedron



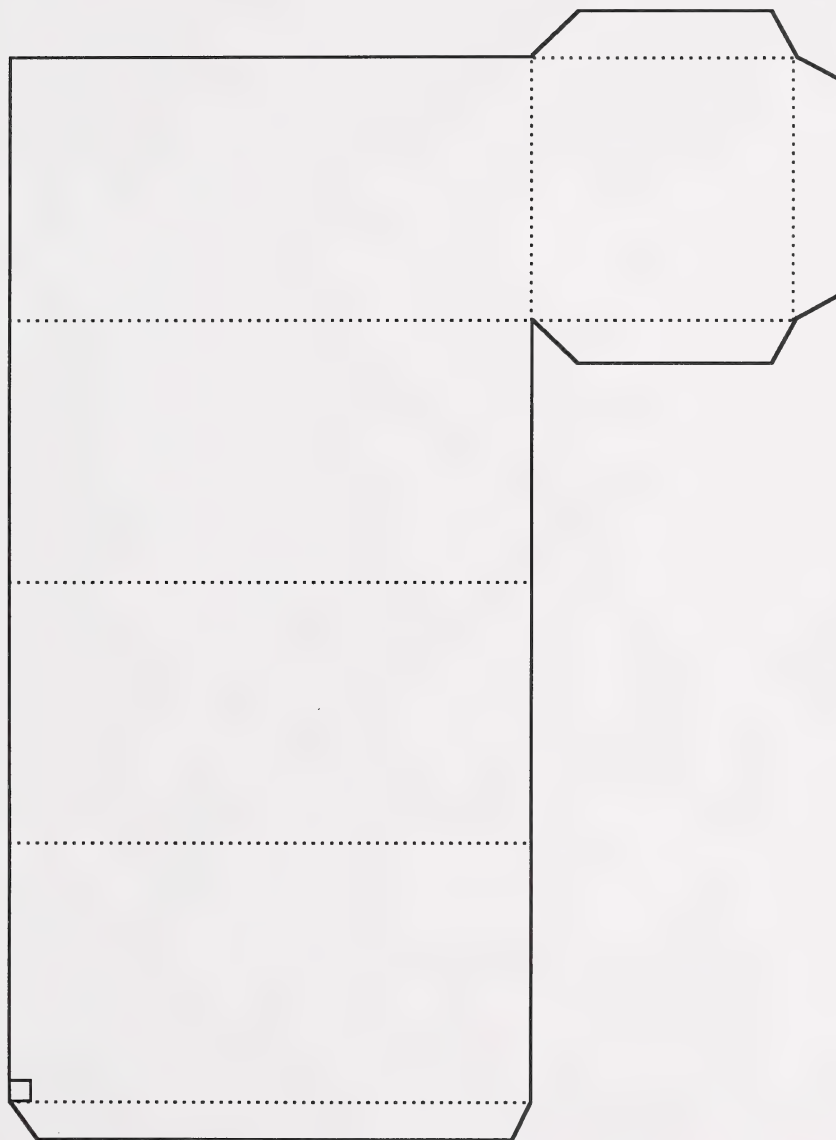
9. Regular Dodecahedron



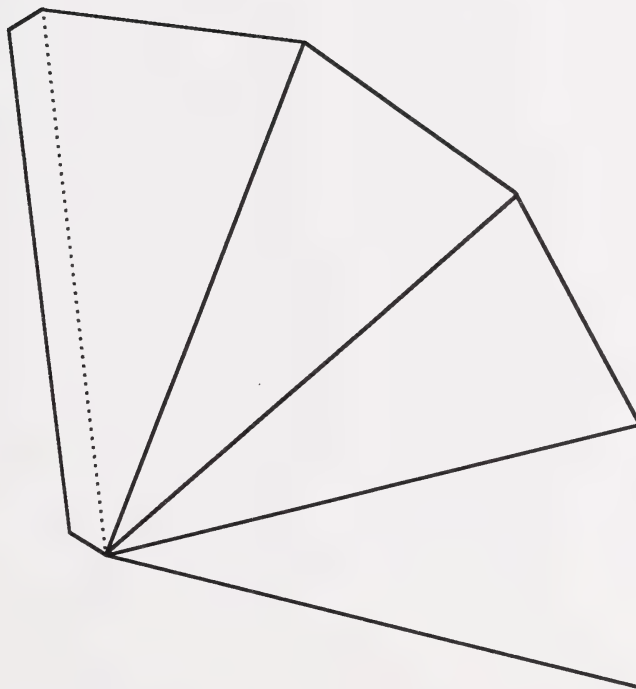
10. Regular Icosahedron



11. Square Prism



12. Square Pyramid



NOTES

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Mathematics 23

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SECOND EDITION
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